Bull Braz Math Soc, New Series (2026) 57:17
https://doi.org/10.1007/s00574-026-00498-3

Splitting Property for Summing Operators

Alexandre C. B. Lima[1] · Joedson Santos[1] · Eduardo B. Silva[2]

Received: 23 October 2025 / Accepted: 28 January 2026

Abstract
In this paper, we prove an abstract version of the splitting property for summing operators, extending the classical result due to A. Pietsch and B. Maurey. Our results provide a technique that may be useful in different contexts, as illustrated in the applications.

Keywords Splitting property · Summing operators

Mathematics Subject Classification 47B10 · 47L20 · 47H60 · 46G25 · 46B25

1 Introduction and Motivation

In the first half of the twentieth century, the theory of continuous linear operators between Banach spaces was already well developed, but it still lacked a refined characterization of those operators that preserve or improve convergence and integrability properties of sequences in the sense of summability. This interest in summability within Banach space theory emerged in the early decades of the second half of the twentieth century and can be traced back to Grothendieck's *Résumé*.

Summability also arises in settings that are not strictly linear, with important applications in Complex Analysis, Combinatorics, Quantum Information Theory, Geometry of Banach Spaces and Classical Inequalities, such as the Hardy–Littlewood and

Alexandre C. B. Lima is supported by Paraíba State Research Foundation (FAPESQ) Grant 2073/2023 Joedson Santos is supported by CNPq Grant 406457/2023-9, Grant 403964/2024-5 and Grant 305655/2025-6 Eduardo B. Silva is supported by CNPq Grant 309230/2023-3.

✉ Joedson Santos
joedson.santos@academico.ufpb.br

Alexandre C. B. Lima
alexandre.cesar@academico.ufpb.br

Eduardo B. Silva
ebsilva@uem.br

1 Departamento de Matemática, Universidade Federal da Paraíba, João Pessoa 58.051-900, Brazil

2 Departamento de Matemática, Universidade Estadual de Maringá, Maringá 87.020-000, Brazil

Published online: 11 March 2026

Bohnenblust–Hille inequalities (see Araújo et al. 2025; Arunachalam et al. 2020; Montanaro 2012; Núñez-Alarcón et al. 2024a; Raposo and Serrano-Rodríguez 2023).

In Grothendieck (1953), Grothendieck introduced ideas that would later be recognized as precursors to the theory of absolutely summing operators. In the following decade, Pietsch (1967), Lindenstrauss and Pełczyński (1968), and Mityagin and Pełczyński (1966) established many of the fundamental properties of these operators. The significant role that absolutely summing linear operators play in operator theory and Banach space theory has motivated the development of analogous notions for classes of summing mappings in various contexts. The nonlinear theory of absolutely summing operators was further advanced by the works of Pietsch (1983), Matos (2003a), Farmer and Johnson (2009), among others. The interest in studying summability in such nonlinear settings has since been explored by numerous authors (see, e.g., Alencar and Matos 1989; Angulo-López and Fernández-Unzueta 2020; Bayart 2018; Belacel et al. 2023; Botelho et al. 2001, 2007, 2019; Çalışkan and Pellegrino 2007; Chávez-Domínguez 2011, 2012; Chen and Zheng 2011, 2012; Dimant 2003; Macedo et al. 2022; Matos 2003b; Núñez-Alarcón et al. 2024b; Pellegrino and Souza 2005, 2006).

If $1 \leq s, r < \infty$, a linear operator $T : X \longrightarrow Y$ between Banach spaces is *absolutely* $(r; s)$*-summing* (or $(r; s)$*-summing*), and we write $T \in \Pi_{r;s}(X; Y)$, if there exists a constant $C > 0$ such that

$$\left(\sum_{i=1}^{m} \|T(x_i)\|^r \right)^{1/r} \leq C \sup_{\varphi \in B_{X^*}} \left(\sum_{i=1}^{m} |\varphi(x_i)|^s \right)^{1/s} \tag{1}$$

for every $x_1, ..., x_m \in X$ and all $m \in \mathbb{N}$, where B_{X^*} denotes the closed unit ball of the topological dual X^* of X, which is a compact Hausdorff space in the weak-star topology (for details, we refer to the classical book Diestel et al. 1995). When $r < s$, one has $\Pi_{r;s}(X; Y) = \{0\}$. Hence, we can assume $s \leq r$. The infimum of all C that satisfy the above inequality defines a norm on $\Pi_{r;s}(X; Y)$, denoted by $\pi_{r;s}(T)$. In addition, $\left(\Pi_{r;s}(X; Y); \pi_{r;s}(\cdot)\right)$ is a Banach space and $\|T\| \leq \pi_{r;s}(T)$ for all $T \in \Pi_{r;s}(X; Y)$. When $s = r$, we write $(\Pi_r(X; Y); \pi_r(\cdot))$ instead of $\left(\Pi_{r;r}(X; Y); \pi_{r;r}(\cdot)\right)$. As is usual, let $\mathcal{L}(X; Y)$ be the space of all continuous linear operators from X to Y. From now on, for any $1 \leq r < \infty$ the symbol r^* denotes the conjugate of r, i.e., $r^* = r/(r-1)$.

The main goal of this paper is to prove an abstract version of the classical *splitting property* for absolutely summing operators. This property appears implicitly in works of Pietsch (1967) and Maurey (1974) (see also Diestel et al. 1995, Lemma 2.23 and Defant and Floret 1993, page 423). It is now known as the *Maurey–Pietsch splitting property*.

Theorem 1.1 (Maurey–Pietsch splitting property) *Let* $1 \leq r < \theta, \delta < \infty$ *with* $\frac{1}{r} = \frac{1}{\theta} + \frac{1}{\delta}$. *If* $T \in \Pi_\delta(X, Y)$, *then given* $m \in \mathbb{N}$ *and* $(x_i)_{i=1}^m \subset X$, *there are* $(\lambda_i)_{i=1}^m \subset \mathbb{K}$ *and* $(y_i)_{i=1}^m \subset Y$ *such that*

$$T(x_i) = \lambda_i y_i \ \textit{for all} \ i = 1, ..., m$$

and

$$\left(\sum_{i=1}^{m}|\lambda_i|^{\delta}\right)^{\frac{1}{\delta}}\sup_{y^*\in B_{Y^*}}\left(\sum_{i=1}^{m}|y^*(y_i)|^{\theta}\right)^{\frac{1}{\theta}}\leq \pi_\delta(u)\sup_{x^*\in B_{X^*}}\left(\sum_{i=1}^{m}|x^*(x_i)|^{r}\right)^{\frac{1}{r}}.$$

When $s = r$, we have $\Pi_r \subset \Pi_\delta$, and thus Theorem 1.1 yields the splitting property for Π_r. However, if $s < r$, then $\Pi_{r;s}$ is not directly comparable to Π_δ, and the splitting property does not follow from the classical result.

This motivates two natural questions:

(i) Does $\Pi_{r;s}$ admit a splitting property?
(ii) Are there nonlinear extensions or generalizations of the splitting property?

In this paper, we answer both questions affirmatively by establishing an abstract framework that includes the linear and nonlinear theories as particular cases.

2 Main Result

Recently, several works have shown that many important results of the theory of summing operators, in fact, require essentially no linear structure to remain valid (see Botelho et al. 2019; Macedo et al. 2022; Pellegrino and Santos 2012; Pellegrino et al. 2012a, b, 2017). The abstract context for our main result was inspired by Pellegrino et al. (2012a).

Let $X_1, ..., X_n, E_1, ..., E_n$ be (arbitrary) nonempty sets such that $E_1 \subseteq X_1, ..., E_n \subseteq X_n$, let Y be a Banach space, and let $\mathcal{H}$ be a family of mappings from $X_1 \times \cdots \times X_n$ to Y. Let also $K_1, ..., K_u$ be compact Hausdorff topological spaces, and let

$$R_l\colon K_l \times X_1 \times \cdots \times X_n \times E_1 \times \cdots \times E_n \longrightarrow [0,\infty),\ l = 1, ..., u$$

be arbitrary maps.

Definition 2.1 Let $1 \leq \alpha_1, ..., \alpha_u, \alpha < \infty$. A mapping $f : X_1 \times \cdots \times X_n \to Y$ of $\mathcal{H}$ is called $R_1, ..., R_u$*-abstract* $(\alpha; \alpha_1, ..., \alpha_u)$*-summing*, and we write $f \in \mathcal{H}_{R_1,...,R_u,(\alpha;\alpha_1,...,\alpha_u)}(X_1, ..., X_n; Y)$, if there is a constant $C > 0$ so that for all $m \in \mathbb{N}$, $x_i^{(k)} \in X_k$, $q_i^{(k)} \in E_k$, with $(i, k, l) \in \{1, ..., m\} \times \{1, ..., n\} \times \{1, ..., u\}$, one has

$$\left(\sum_{i=1}^{m}\left\|f\left(x_i^{(1)},\ldots,x_i^{(n)}\right)-f\left(q_i^{(1)},\ldots,q_i^{(n)}\right)\right\|^{\alpha}\right)^{\frac{1}{\alpha}}$$
$$\leq C\cdot\prod_{l=1}^{u}\sup_{\varphi\in K_l}\left(\sum_{i=1}^{m}R_l\left(\varphi,x_i^{(1)},...,x_i^{(n)},q_i^{(1)},...,q_i^{(n)}\right)^{\alpha_l}\right)^{\frac{1}{\alpha_l}}. \tag{2}$$

The infimum of all C for which (2) holds is denoted by $\|f\|_{R_1,...,R_u,(\alpha;\alpha_1,...,\alpha_u)}$ and also satisfies the corresponding inequality.

Our goal is to show that this abstract class of summing operators satisfies the splitting property. The key tool in our argument is the following inequality, first used by Pietsch in (1967, Theorem 4) (see also Defant and Floret 1993, Theorem 11.5, Pietsch 1987, Theorem 1.3.10) and presented in the form below in Popa (2017, Proposition 2).

Lemma 2.2 *Let Y be a Banach space, and suppose that $1 \leq \alpha, \theta, \delta < \infty$ are such that $\frac{1}{\alpha} = \frac{1}{\theta} + \frac{1}{\delta}$. For every $(\beta_1, ..., \beta_m) \in [0, \infty)^m$, $(y_1, ..., y_m) \in Y^m$, and $(\lambda_1, ..., \lambda_m) \in (0, \infty)^m$, we have*

$$\left\| \sum_{i=1}^{m} \frac{\beta_i y_i}{\lambda_i} \right\| \leq \left(\sum_{i=1}^{m} \frac{\beta_i^{\theta^*} \|y_i\|^\alpha}{\lambda_i^\delta} \right)^{\frac{1}{\delta}} \left(\sum_{i=1}^{m} \|y_i\|^\alpha \right)^{\frac{1}{\theta}} \left(\sum_{i=1}^{m} \beta_i^{\theta^*} \right)^{\frac{1}{\alpha^*}} .$$

Theorem 2.3 *Let $1 \leq \alpha_1, ..., \alpha_u, \alpha, \theta, \delta < \infty$ satisfy $\alpha < \theta, \delta < \infty$ and $\frac{1}{\alpha} = \frac{1}{\theta} + \frac{1}{\delta}$. If $f \in \mathcal{H}_{R_1,...,R_u,(\alpha;\alpha_1,...,\alpha_u)}(X_1, ..., X_n; Y)$, then given $m \in \mathbb{N}$, $x_i^{(k)} \in X_k$, $q_i^{(k)} \in E_k$, with $(i, k) \in \{1, ..., m\} \times \{1, ..., n\}$, there are $(\lambda_i)_{i=1}^m \subset \mathbb{K}$ and $(y_i)_{i=1}^m \subset Y$ such that*

$$f\left(x_i^{(1)}, \ldots, x_i^{(n)}\right) - f\left(q_i^{(1)}, \ldots, q_i^{(n)}\right) = \lambda_i y_i \ \textit{for all} \ \ i = 1, ..., m$$

and

$$\left(\sum_{i=1}^{m} |\lambda_i|^\delta \right)^{\frac{1}{\delta}} \sup_{y^* \in B_{Y^*}} \left(\sum_{i=1}^{m} |y^*(y_i)|^\theta \right)^{\frac{1}{\theta}}$$
$$\leq \|f\|_{R_1,...,R_u,(\alpha;\alpha_1,...,\alpha_u)} \cdot \prod_{l=1}^{u} \sup_{\varphi \in K_l} \left(\sum_{i=1}^{m} R_l\left(\varphi, x_i^{(1)}, ..., x_i^{(n)}, q_i^{(1)}, ..., q_i^{(n)}\right)^{\alpha_l} \right)^{\frac{1}{\alpha_l}} .$$

Proof For each $i = 1, ..., m$, define

$$\lambda_i = \left\| f\left(x_i^{(1)}, \ldots, x_i^{(n)}\right) - f\left(q_i^{(1)}, \ldots, q_i^{(n)}\right) \right\|^{\frac{\alpha}{\delta}} \tag{3}$$

and

$$y_i = \begin{cases} \dfrac{f\left(x_i^{(1)}, \ldots, x_i^{(n)}\right) - f\left(q_i^{(1)}, \ldots, q_i^{(n)}\right)}{\lambda_i} & \text{if } \lambda_i \neq 0 \\ 0 & \text{if } \lambda_i = 0. \end{cases}$$

Note that, if $\lambda_i \neq 0$, then $f\left(x_i^{(1)}, \ldots, x_i^{(n)}\right) - f\left(q_i^{(1)}, \ldots, q_i^{(n)}\right) = \lambda_i y_i$. Now, if $\lambda_i = 0$, it follows from (3) that $f\left(x_i^{(1)}, \ldots, x_i^{(n)}\right) - f\left(q_i^{(1)}, \ldots, q_i^{(n)}\right) = 0 = \lambda_i y_i$. Moreover, since f is $R_1, ..., R_u$-abstract $(\alpha; \alpha_1, ..., \alpha_u)$-summing, we have

$$\sum_{i=1}^{m} |\lambda_i|^\delta = \sum_{i=1}^{m} \left\| f\left(x_i^{(1)}, \ldots, x_i^{(n)}\right) - f\left(q_i^{(1)}, \ldots, q_i^{(n)}\right) \right\|^\alpha$$
$$\leq \|f\|^\alpha_{R_1,...,R_u,(\alpha;\alpha_1,...,\alpha_u)} \cdot \left(\prod_{l=1}^{u} \sup_{\varphi \in K_l} \left(\sum_{i=1}^{m} R_l\left(\varphi, x_i^{(1)}, ..., x_i^{(n)}, q_i^{(1)}, ..., q_i^{(n)}\right)^{\alpha_l} \right)^{\frac{1}{\alpha_l}} \right)^\alpha. \quad (4)$$

Considering $I = \{i : \lambda_i \neq 0\}$ we have that

$$\sup_{y^* \in B_{Y^*}} \left(\sum_{i=1}^{m} |y^*(y_i)|^\theta \right)^{\frac{1}{\theta}} = \sup_{y^* \in B_{Y^*}} \left(\sum_{i \in I} |y^*(y_i)|^\theta \right)^{\frac{1}{\theta}}.$$

Without loss of generality, we can assume that $I = \{1, ..., m\}$. So, by Pietsch (1987, Lemma 1.1.14) we have

$$\sup_{y^* \in B_{Y^*}} \left(\sum_{i=1}^{m} |y^*(y_i)|^\theta \right)^{\frac{1}{\theta}} = \sup_{(\beta_i)_{i=1}^m \in B_{\ell^m_{\theta^*}}} \left\| \sum_{j=1}^{m} \beta_i y_i \right\|.$$

Thus, for $(\beta_j)_{j=1}^m \in B_{\ell^m_{\theta^*}}$, it follows from Lemma 2.2 and (3) that

$$\left\| \sum_{i=1}^{m} \beta_i y_i \right\| = \left\| \sum_{i=1}^{m} \frac{\beta_i \left(f\left(x_i^{(1)}, \ldots, x_i^{(n)}\right) - f\left(q_i^{(1)}, \ldots, q_i^{(n)}\right) \right)}{\lambda_i} \right\|$$
$$\leq \left(\sum_{i=1}^{m} \frac{|\beta_i|^{\theta^*} \left\| f\left(x_i^{(1)}, \ldots, x_i^{(n)}\right) - f\left(q_i^{(1)}, \ldots, q_i^{(n)}\right) \right\|^\alpha}{\lambda_i^\delta} \right)^{\frac{1}{\delta}}$$
$$\cdot \left(\sum_{i=1}^{m} \left\| f\left(x_i^{(1)}, \ldots, x_i^{(n)}\right) - f\left(q_i^{(1)}, \ldots, q_i^{(n)}\right) \right\|^\alpha \right)^{\frac{1}{\theta}} \cdot \left(\sum_{i=1}^{m} |\beta_i|^{\theta^*} \right)^{\frac{1}{\alpha^*}}$$
$$\leq \left(\sum_{i=1}^{m} \left\| f\left(x_i^{(1)}, \ldots, x_i^{(n)}\right) - f\left(q_i^{(1)}, \ldots, q_i^{(n)}\right) \right\|^\alpha \right)^{\frac{1}{\theta}}.$$

Since f is $R_1, ..., R_u$-abstract $(\alpha; \alpha_1, ..., \alpha_u)$-summing, we obtain

$$\left\| \sum_{i=1}^{m} \beta_i y_i \right\| \leq \|f\|^{\frac{\alpha}{\theta}}_{R_1,...,R_u,(\alpha;\alpha_1,...,\alpha_u)} \cdot \left(\prod_{l=1}^{u} \sup_{\varphi \in K_l} \left(\sum_{i=1}^{m} R_l\left(\varphi, x_i^{(1)}, ..., x_i^{(n)}, q_i^{(1)}, ..., q_i^{(n)}\right)^{\alpha_l} \right)^{\frac{1}{\alpha_l}} \right)^{\frac{\alpha}{\theta}}.$$

This yields

$$\sup_{y^*\in B_{Y^*}}\left(\sum_{i=1}^{m}|y^*(y_i)|^{\theta}\right)^{\frac{1}{\theta}}$$

$$\leq \|f\|_{R_1,\ldots,R_u,(\alpha;\alpha_1,\ldots,\alpha_u)}^{\frac{\alpha}{\theta}}\cdot\left(\prod_{l=1}^{u}\sup_{\varphi\in K_l}\left(\sum_{i=1}^{m}R_l\left(\varphi,x_i^{(1)},\ldots,x_i^{(n)},q_i^{(1)},\ldots,q_i^{(n)}\right)^{\alpha_l}\right)^{\frac{1}{\alpha_l}}\right)^{\frac{\alpha}{\theta}}. \quad (5)$$

From (4), (5) and $\frac{1}{\alpha}=\frac{1}{\theta}+\frac{1}{\delta}$ it follows the result. □

If there exists $a=\left(a^{(1)},\ldots,a^{(n)}\right)\in E_1\times\cdots\times E_n$ such that $f(a)=0$, then by taking $\left(q_i^{(1)},\ldots,q_i^{(n)}\right)=a$ for all $i\in\{1,\ldots,m\}$ in Theorem 2.3, we obtain an abstract generalization of the splitting property.

Corollary 2.4 (Abstract Splitting Property) *Let* $1\leq\alpha_1,\ldots,\alpha_u,\alpha,\theta,\delta<\infty$ *with* $\alpha<\theta,\delta<\infty$ *and* $\frac{1}{\alpha}=\frac{1}{\theta}+\frac{1}{\delta}$. *Suppose that there exists* $a=\left(a^{(1)},\ldots,a^{(n)}\right)\in E_1\times\cdots\times E_n$ *such that* $f(a)=0$. *If* $f\in\mathcal{H}_{R_1,\ldots,R_u,(\alpha;\alpha_1,\ldots,\alpha_u)}(X_1,\ldots,X_n;Y)$, *then given* $m\in\mathbb{N}$, $x_i^{(k)}\in X_k$, *with* $(i,k)\in\{1,\ldots,m\}\times\{1,\ldots,n\}$, *there are* $(\lambda_i)_{i=1}^m\subset\mathbb{K}$ *and* $(y_i)_{i=1}^m\subset Y$ *such that*

$$f\left(x_i^{(1)},\ldots,x_i^{(n)}\right)=\lambda_i y_i \text{ for all } i=1,\ldots,m$$

and

$$\left(\sum_{i=1}^{m}|\lambda_i|^{\delta}\right)^{\frac{1}{\delta}}\sup_{y^*\in B_{Y^*}}\left(\sum_{i=1}^{m}|y^*(y_i)|^{\theta}\right)^{\frac{1}{\theta}}$$

$$\leq\|f\|_{R_1,\ldots,R_u,(\alpha;\alpha_1,\ldots,\alpha_u)}\cdot\prod_{l=1}^{u}\sup_{\varphi\in K_l}\left(\sum_{i=1}^{m}R_l\left(\varphi,x_i^{(1)},\ldots,x_i^{(n)},a^{(1)},\ldots,a^{(n)}\right)^{\alpha_l}\right)^{\frac{1}{\alpha_l}}.$$

3 Applications

Now we will apply our main result to obtain the splitting property for several classes of summing operators (linear and nonlinear).

3.1 Absolutely Summing Linear Operators

Let X and Y be Banach spaces and $1\leq s\leq r<\infty$. Choosing the following parameters in the abstract framework of Sect. 2:

$$\begin{cases} u = n = 1 \\ \alpha = r \text{ and } \alpha_1 = s \\ X_1 = X \text{ and } E_1 = \{0\} \subset X \\ K_1 = B_{X^*} \\ \mathcal{H} = \mathcal{L}(X; Y) \\ R_1(\varphi, x, 0) = |\varphi(x)| \,. \end{cases}$$

With these choices, we have that a linear mapping $T \in \mathcal{L}(X; Y)$ is R_1-abstract $(r; s)$ -summing precisely when T is absolutely $(r; s)$-summing. Thus, Corollary 2.4 ensures the splitting property for the class of $(r; s)$-summing operators.

Corollary 3.1 *Suppose that* $1 \le s \le r < \theta, \delta < \infty$ *such that* $\frac{1}{r} = \frac{1}{\theta} + \frac{1}{\delta}$, *and let* $T \in \Pi_{r;s}(X; Y)$. *Then, given* $m \in \mathbb{N}$ *and* $(x_i)_{i=1}^m \subset X$, *there are* $(\lambda_i)_{i=1}^m \subset \mathbb{K}$ *and* $(y_i)_{i=1}^m \subset Y$ *such that*

$$T\,(x_i) = \lambda_i y_i \ \textit{for all} \ \ i = 1, ..., m$$

and

$$\left(\sum_{i=1}^m |\lambda_i|^\delta\right)^{\frac{1}{\delta}} \sup_{y^* \in B_{Y^*}} \left(\sum_{i=1}^m |y^*\,(y_i)\,|^\theta\right)^{\frac{1}{\theta}} \le \pi_{r;s}(T) \sup_{\varphi \in B_{X^*}} \left(\sum_{i=1}^m |\varphi\,(x_i)\,|^s\right)^{\frac{1}{s}}.$$

3.2 Lipschitz (r, s)-Summing Operators

Let X be a pointed metric space with its base point denoted by 0, and let Y be a Banach space. We denote by $Lip_0(X; Y)$ the subspace of the Lipschitz operators such that $T(0) = 0$, endowed with the Lipschitz norm defined by

$$Lip(T) = \sup\left\{\frac{\|T(x) - T(y)\|}{d(x, y)} \ : \ x, y \in X, \ x \ne y\right\}.$$

It is known that $(Lip_0(X; Y); Lip(\cdot))$ is a Banach space, and when $Y = \mathbb{K}$, we write $X^\# = Lip_0(X; \mathbb{K})$. Let $1 \le s \le r < \infty$. A mapping $T \in Lip_0(X; Y)$ is called *Lipschitz* $(r; s)$*-summing* if there is a constant $C > 0$ such that

$$\left(\sum_{i=1}^m \|T(x_i) - T(q_i)\|^r\right)^{\frac{1}{r}} \le C \sup_{\varphi \in B_{X^\#}} \left(\sum_{i=1}^m |\varphi(x_i) - \varphi(q_i)|^s\right)^{\frac{1}{s}} \tag{6}$$

for all $m \in \mathbb{N}$ and $x_i \in X, i = 1, \ldots, m$. This class is denoted by $\Pi_{r;s}^L$ (when $r = s$ we write Π_r^L) and $\pi_{r;s}^L(T)$ (or $\pi_r^L(T)$ when $s = r$) is the smallest number C that satisfies (6).

The class of Lipschitz $(r; s)$-summing operators is a natural generalization of the class of Lipschitz r-summing operators introduced by Farmer and Johnson (2009).

Let us choose the following parameters in the abstract framework of Sect. 2:

$$\begin{cases} u = n = 1 \\ \alpha = r \text{ and } \alpha_1 = s \\ E_1 = X_1 = X \\ K_1 = B_{X^\#} \\ \mathcal{H} = Lip_0(X;Y) \\ R_1(\varphi, x, q) = |\varphi(x) - \varphi(q)|\,. \end{cases}$$

With these choices we have that a Lipschitz mapping $T \in Lip_0(X;Y)$ is R_1-abstract $(r;s)$ -summing if and only if T is Lipschitz $(r;s)$-summing. Since $T(0) = 0$, taking $a_1 = \cdots = a_n = 0$, the splitting property for the class of Lipschitz $(r;s)$-summing operators follows from Corollary 2.4.

Corollary 3.2 *Suppose that* $1 \le s \le r < \theta, \delta < \infty$ *such that* $\frac{1}{r} = \frac{1}{\theta} + \frac{1}{\delta}$, *and let* $T \in \Pi^L_{r;s}(X;Y)$. *Then, given* $m \in \mathbb{N}$ *and* $(x_i)_{i=1}^m \subset X$, *there are* $(\lambda_i)_{i=1}^m \subset \mathbb{K}$ *and* $(y_i)_{i=1}^m \subset Y$ *such that*

$$T(x_i) = \lambda_i y_i \ \text{ for all } i = 1, ..., m$$

and

$$\left(\sum_{i=1}^m |\lambda_i|^\delta\right)^{\frac{1}{\delta}} \sup_{y^* \in B_{Y^*}} \left(\sum_{i=1}^m |y^*(y_i)|^\theta\right)^{\frac{1}{\theta}} \le \pi^L_{r;s}(T) \sup_{\varphi \in B_{X^\#}} \left(\sum_{i=1}^m |\varphi(x_i)|^s\right)^{\frac{1}{s}}.$$

3.3 $(r; s; G)$-Summing Linear Operators

If X and G are Banach spaces, then $X \otimes_\varepsilon G$ denotes the injective tensor product, and we denote by $X \hat{\otimes}_\varepsilon G$ the completion of $X \otimes_\varepsilon G$. Kislyakov (1992) introduced the following notion in the setting of injective tensor products: Given $1 \le s \le r < \infty$ and three Banach spaces X, G, and Y, a linear operator T from $X \hat{\otimes}_\varepsilon G$ into Y is called $(r;s;G)$-*summing* if there is a constant $C > 0$ such that for all $m \in \mathbb{N}$ and $v_i \in X \otimes_\varepsilon G$, $i = 1, \ldots, m$, we have

$$\left(\sum_{i=1}^m \|T(v_i)\|^r\right)^{\frac{1}{r}} \le C \sup_{\varphi \in B_{X^*}} \left(\sum_{i=1}^m \|v_i(\varphi)\|_G^s\right)^{\frac{1}{s}}$$

where $v_i(\varphi) = \sum_{k=1}^{n_k} \varphi(x_{k,i}) g_{k,i}$ for $v_i = \sum_{k=1}^{n_k} x_{k,i} \otimes g_{k,i}$ for some $x_{k,i} \in X$ and $g_{k,i} \in G$. The space of such operators is denoted by $\Pi^G_{r;s}(X \hat{\otimes}_\varepsilon G; Y)$, and $\|T\|^G_{r;s}$ is the infimum being taken over all the constants C above.

Choosing the following parameters, in the abstract framework of Sect. 3,

$$\begin{cases} u = n = 1 \\ \alpha = r \text{ and } \alpha_1 = s \\ X_1 = X\hat{\otimes}_\varepsilon G \text{ and } E_1 = \{0\} \subset X_1 \\ K_1 = B_{X^*} \\ \mathcal{H} = \mathcal{L}(X\hat{\otimes}_\varepsilon G; Y) \\ R_1(\varphi, v, 0) = \|v(\varphi)\|_G \, , \end{cases}$$

we conclude that a linear mapping $T \in \mathcal{L}(X\hat{\otimes}_\varepsilon G; Y)$ is R_1-abstract $(r; s)$ -summing precisely when T is $(r; s; G)$-summing. By Corollary 2.4, we have the splitting property for this class of operators.

Corollary 3.3 *Suppose that $1 \leq s \leq r < \theta, \delta < \infty$ such that $\frac{1}{r} = \frac{1}{\theta} + \frac{1}{\delta}$, and let $T \in \Pi^G_{r;s}(X\hat{\otimes}_\varepsilon G; Y)$. Then, given $m \in \mathbb{N}$ and $(v_i)_{i=1}^m \subset X\hat{\otimes}_\varepsilon G$, there are $(\lambda_i)_{i=1}^m \subset \mathbb{K}$ and $(y_i)_{i=1}^m \subset Y$ such that*

$$T\,(v_i) = \lambda_i y_i \ \textit{ for all } i = 1, ..., m$$

and

$$\left(\sum_{i=1}^m |\lambda_i|^\delta\right)^{\frac{1}{\delta}} \sup_{y^* \in B_{Y^*}} \left(\sum_{i=1}^m |y^*\,(y_i)\,|^\theta\right)^{\frac{1}{\theta}} \leq \|T\|^G_{r;s} \sup_{\varphi \in B_{X^*}} \left(\sum_{i=1}^m \|v_i\,(\varphi)\,\|^s\right)^{\frac{1}{s}} .$$

3.4 $(\ell_s^{strong}; \ell_r)$-Summing Linear Operators

Let $1 \leq s \leq r < \infty$ and X, Y, and Z be Banach spaces. A bounded linear operator T from $\mathcal{L}(X^*, Z)$ into Y is called $(\ell_s^{strong}; \ell_r)$-*summing* if there is a constant $C > 0$ such that

$$\left(\sum_{i=1}^m \|T(v_i)\|^r\right)^{\frac{1}{r}} \leq C \sup_{\varphi \in B_{X^*}} \left(\sum_{i=1}^m \|v_i(\varphi)\|_Z^s\right)^{\frac{1}{s}}$$

for all $m \in \mathbb{N}$ and $v_i \in \mathcal{L}(X^*, Z)$, with $i = 1, \ldots, m$. The space of such operators is denoted by $\Pi_{(\ell_s^{strong};\ell_r)}(\mathcal{L}(X^*, Z); Y)$ and $\|T\|_{(\ell_s^{strong};\ell_r)}$ is the infimum being taken over all the constant C. The concept of the $(\ell_r^{strong}; \ell_r)$-summing linear operators was introduced by Blasco and Signes (2003).

Choosing the following parameters in the abstract framework of Sect. 2:

$$\begin{cases} u = n = 1 \\ \alpha = r \text{ and } \alpha_1 = s \\ X_1 = \mathcal{L}(X^*, Z) \text{ and } E_1 = \{0\} \subset X_1 \\ K_1 = B_{X^*} \\ \mathcal{H} = \mathcal{L}(\mathcal{L}(X^*, Z); Y) \\ R_1(\varphi, v, 0) = \|v(\varphi)\|_Z \, . \end{cases}$$

With these choices we have that a linear mapping $T \in \mathcal{L}(\mathcal{L}(X^*, Z); Y)$ is R_1-abstract $(r; s)$ -summing precisely when T is $(\ell_s^{strong}; \ell_r)$-summing. By Corollary 2.4, we obtain the splitting property for this class of operators.

Corollary 3.4 *Suppose that* $1 \leq s \leq r < \theta, \delta < \infty$ *such that* $\frac{1}{r} = \frac{1}{\theta} + \frac{1}{\delta}$, *and let* $T \in \Pi_{(\ell_s^{strong}; \ell_r)}(\mathcal{L}(X^*, Z); Y)$. *Then, given* $m \in \mathbb{N}$ *and* $(v_i)_{i=1}^m \subset \mathcal{L}(X^*, Z)$, *there are* $(\lambda_i)_{i=1}^m \subset \mathbb{K}$ *and* $(y_i)_{i=1}^m \subset Y$ *such that*

$$T(v_i) = \lambda_i y_i \text{ for all } i = 1, ..., m$$

and

$$\left(\sum_{i=1}^{m} |\lambda_i|^{\delta}\right)^{\frac{1}{\delta}} \sup_{y^* \in B_{Y^*}} \left(\sum_{i=1}^{m} |y^*(y_i)|^{\theta}\right)^{\frac{1}{\theta}} \leq \|T\|_{(\ell_s^{strong}; \ell_r)} \sup_{\varphi \in B_{X^*}} \left(\sum_{i=1}^{m} \|v_i(\varphi)\|^{s}\right)^{\frac{1}{s}}.$$

3.5 Absolutely Summing Multilinear Operators

Let $X_1, ..., X_n, Y$ be Banach spaces and $1 \leq r, r_1, \ldots, r_n < \infty$, such that $\frac{1}{r} \leq \frac{1}{r_1} + \cdots + \frac{1}{r_n}$. A mapping $T \in \mathcal{L}(X_1, ..., X_n; Y)$ is called *absolutely* $(r; r_1, \ldots, r_n)$-*summing* if there is a constant $C > 0$ such that

$$\left(\sum_{i=1}^{m} \left\|T(x_i^{(1)}, \ldots, x_i^{(n)})\right\|^{r}\right)^{\frac{1}{r}} \leq C \prod_{l=1}^{n} \sup_{\varphi \in B_{X_l^*}} \left(\sum_{i=1}^{m} \left|\varphi(x_i^{(l)})\right|^{r_l}\right)^{\frac{1}{r_l}} \tag{7}$$

for all $m \in \mathbb{N}$ and $x_i^{(l)} \in X_l$, $i = 1, \ldots, m$, $l = 1, \ldots, n$. This class is denoted by $\mathcal{L}_{as,(r;r_1,\ldots,r_n)}(X_1, ..., X_n; Y)$. When $X_1 = \cdots = X_n = X$ we write $\mathcal{L}_{as,(r;r_1,\ldots,r_n)}(^n X; Y)$ and when $r_1 = \cdots = r_n = s$ we write $\mathcal{L}_{as,(r;s)}(X_1, ..., X_n; Y)$.

The definition of absolutely $(r; r_1, \ldots, r_n)$-summing n-linear forms is due to Pietsch Pietsch (1983). In Alencar and Matos (1989), Matos presented a definition for vector-valued mappings.

Now, let us choose the following parameters in the abstract framework of Sect. 2:

$$\begin{cases} u = n \\ \alpha = r \text{ and } \alpha_l = r_l \text{ for every } l = 1, \ldots, n \\ K_l = B_{X_l^*} \\ E_l = \{0\} \subset X_l \\ \mathcal{H} = \mathcal{L}(X_1, ..., X_n; Y) \\ R_l(\varphi, x^{(1)}, \ldots, x^{(n)}, 0, ..., 0) = \left|\varphi(x^{(l)})\right|. \end{cases}$$

With these choices, it follows from (7) and (2) that an n-linear mapping $T \in \mathcal{L}(X_1, ..., X_n; Y)$ is $R_1, \ldots, R_n$-abstract $(r; r_1, \ldots, r_n)$ -summing if, and only if, T is absolutely $(r; r_1, \ldots, r_n)$-summing.

Corollary 2.4 ensures the splitting property for the absolutely $(r; r_1, \ldots, r_n)$-summing operators.

Corollary 3.5 *Suppose that* $1 \leq r_1, \ldots, r_n \leq r < \theta, \delta < \infty$ *such that* $\frac{1}{r} = \frac{1}{\theta} + \frac{1}{\delta}$, *and let* $T \in \mathcal{L}_{as,(r;r_1,\ldots,r_n)}(X_1, ..., X_n; Y)$. *Then, given* $m \in \mathbb{N}$ *and* $(x_i^{(k)})_{i=1}^m \subset X_k$, *with* $k = 1, ..., n$, *there are* $(\lambda_i)_{i=1}^m \subset \mathbb{K}$ *and* $(y_i)_{i=1}^m \subset Y$ *such that*

$$T\left(x_i^{(1)}, \ldots, x_i^{(n)}\right) = \lambda_i y_i \ \textit{for all } i = 1, ..., m$$

and

$$\left(\sum_{i=1}^m |\lambda_i|^\delta\right)^{\frac{1}{\delta}} \sup_{y^* \in B_{Y^*}} \left(\sum_{i=1}^m |y^*(y_i)|^\theta\right)^{\frac{1}{\theta}} \leq \|T\|_{as(r;r_1,\ldots,r_n)} \prod_{l=1}^n \sup_{\varphi \in B_{X_l^*}} \left(\sum_{i=1}^m \left|\varphi(x_i^{(l)})\right|^{r_l}\right)^{\frac{1}{r_l}}.$$

3.6 Lipschitz (*r*; *s*)-Summing Multilinear Operators

Let $X_1, ..., X_n, Y$ be Banach spaces and $1 \leq s \leq r < \infty$. An n-linear operator $T : X_1 \times \cdots \times X_n \longrightarrow Y$ is *n-Lipschitz* $(r; s)$*-summing* ($T \in \mathcal{L}^{nLip}_{(r;s)}(X_1, ..., X_n; Y)$) if there exists a constant $C \geq 0$ such that

$$\begin{aligned}&\left(\sum_{i=1}^m \| T(x_i^{(1)}, ..., x_i^{(n)}) - T(q_i^{(1)}, ..., q_i^{(n)}) \|^r\right)^{1/r} \\ &\leq C \sup_{\phi \in B_{\mathcal{L}(X_1,\ldots,X_n;\mathbb{K})}} \left(\sum_{i=1}^m |\phi(x_i^{(1)}, ..., x_i^{(n)}) - \phi(q_i^{(1)}, ..., q_i^{(n)})|^s\right)^{1/s} \qquad (8)\end{aligned}$$

for every $m \in \mathbb{N}$, $x_i^{(k)}, q_i^{(k)} \in X_k$ with $k = 1, ..., n$ and $i = 1, ..., m$ In this case, $\|T\|_{nLip(r;s)}$ is the smallest number C that satisfies this inequality. The notion of a Lipschitz $(r; r)$-summing multilinear mapping was introduced by Angulo-López and Fernández-Unzueta (2020).

Choosing the following parameters in the abstract framework of Sect. 2,

$$\begin{cases} u = 1 \\ \alpha = r \text{ and } \alpha_1 = s \\ K_1 = B_{(X_1 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_n)^*} \\ E_l = X_l \text{ for every } l = 1, \ldots, n \\ \mathcal{H} = \mathcal{L}(X_1, ..., X_n; Y) \\ R_1(\phi, x^{(1)}, \ldots, x^{(n)}, q^{(1)}, \ldots, q^{(n)}) = \left|\phi(x^{(1)} \otimes ... \otimes x^{(n)}) - \phi(q^{(1)} \otimes ... \otimes q^{(n)})\right|, \end{cases}$$

we conclude that an n-linear mapping $T \in \mathcal{L}(X_1, ..., X_n; Y)$ is R_1-abstract $(r; s)$-summing if and only if T is n-Lipschitz $(r; s)$-summing. Thus, Corollary 2.4 ensures:

Corollary 3.6 *Suppose that* $1 \leq s \leq r < \theta, \delta < \infty$ *such that* $\frac{1}{r} = \frac{1}{\theta} + \frac{1}{\delta}$, *and let* $T \in \mathcal{L}^{nLip}_{(r;s)}(X_1, ..., X_n; Y)$. *Then, given* $m \in \mathbb{N}$ *and* $(x_i^{(k)})_{i=1}^m \subset X_k$, *with* $k = 1, ..., n$, *there are* $(\lambda_i)_{i=1}^m \subset \mathbb{K}$ *and* $(y_i)_{i=1}^m \subset Y$ *such that*

$$T\left(x_i^{(1)}, \ldots, x_i^{(n)}\right) = \lambda_i y_i \ \textit{ for all } i = 1, ..., m$$

and

$$\left(\sum_{i=1}^m |\lambda_i|^\delta\right)^{\frac{1}{\delta}} \sup_{y^* \in B_{Y^*}} \left(\sum_{i=1}^m |y^*(y_i)|^\theta\right)^{\frac{1}{\theta}} \\ \leq \|T\|_{nLip(r;s)} \sup_{\phi \in B_{\mathcal{L}(X_1,...,X_n;\mathbb{K})}} \left(\sum_{i=1}^m |\phi(x_i^{(1)}, ..., x_i^{(n)})|^s\right)^{1/s}.$$

3.7 Strongly Summing Multilinear Operators

Let $X_1, ..., X_n, Y$ be Banach spaces. If $1 \leq s \leq r < \infty$, $T \in \mathcal{L}(X_1, ..., X_n; Y)$ is *strongly* $(r;s)$*-summing*$(T \in \mathcal{L}_{ss,(r;s)}(X_1, ..., X_n; Y))$ if there exists a constant $C \geq 0$ such that

$$\left(\sum_{i=1}^m \| T(x_i^{(1)}, ..., x_i^{(n)}) \|^r\right)^{1/r} \leq C \sup_{\phi \in B_{\mathcal{L}(X_1,...,X_n;\mathbb{K})}} \left(\sum_{i=1}^m |\phi(x_i^{(1)}, ..., x_i^{(n)})|^s\right)^{1/s}$$

for every $m \in \mathbb{N}$, $x_i^{(k)} \in X_k$ with $k = 1, ..., n$ and $i = 1, ..., m$. In this case, $\|T\|_{ss(r;s)}$ is the smallest number C that satisfies this inequality.

This class was introduced by Botelho et al. (2008) as a natural generalization of strongly $(r;r)$-summing multilinear operators due to Dimant (2003). This class may be the one that best translates the properties of the original linear concept into the multilinear setting.

For $1 \leq s \leq r < \infty$, let us choose the following parameters in the abstract framework of Sect. 2:

$$\begin{cases} u = 1 \\ \alpha = r \ \text{ and } \alpha_1 = s \\ K_1 = B_{(X_1 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_n)^*} \\ E_l = \{0\} \subset X_l \text{ for every } l = 1, \ldots, n \\ \mathcal{H} = \mathcal{L}(X_1, ..., X_n; Y) \\ R_1(\phi, x^{(1)}, \ldots, x^{(n)}, 0, ..., 0) = \left|\phi(x^{(1)} \otimes \cdots \otimes x^{(n)})\right|. \end{cases}$$

With these choices we have that an n-linear mapping $T \in \mathcal{L}(X_1, ..., X_n; Y)$ is R_1-abstract $(r;s)$ -summing precisely when T is strongly $(r;s)$-summing. Thus, Corollary 2.4 gives us the following:

Corollary 3.7 *Suppose that* $1 \leq s \leq r < \theta, \delta < \infty$ *such that* $\frac{1}{r} = \frac{1}{\theta} + \frac{1}{\delta}$*, and let* $T \in \mathcal{L}_{ss,(r;s)}(X_1, ..., X_n; Y)$*. Then, given* $m \in \mathbb{N}$ *and* $(x_i^{(k)})_{i=1}^m \subset X_k$*, with* $k = 1, ..., n$*,* *there are* $(\lambda_i)_{i=1}^m \subset \mathbb{K}$ *and* $(y_i)_{i=1}^m \subset Y$ *such that*

$$T\left(x_i^{(1)}, \ldots, x_i^{(n)}\right) = \lambda_i y_i \ \textit{for all } i = 1, ..., m$$

and

$$\left(\sum_{i=1}^m |\lambda_i|^\delta\right)^{\frac{1}{\delta}} \sup_{y^* \in B_{Y^*}} \left(\sum_{i=1}^m |y^*(y_i)|^\theta\right)^{\frac{1}{\theta}} \\ \leq \|T\|_{ss(r;s)} \sup_{\phi \in B_{\mathcal{L}(X_1,...,X_n;\mathbb{K})}} \left(\sum_{i=1}^m |\phi(x_i^{(1)}, ..., x_i^{(n)})|^s\right)^{1/s}.$$

3.8 Semi-integral Multilinear Operators

Let $X_1, ..., X_n, Y$ be Banach spaces and $1 \leq s \leq r < \infty$. An n-linear mapping $T \in \mathcal{L}(X_1, ...Y_n; Y)$ is $(r; s)$-*semi-integral* ($T \in \mathcal{L}_{si,(r;s)}(X_1, ..., X_n; Y)$) if there exists a $C \geq 0$ such that

$$\left(\sum_{i=1}^m \| T(x_i^{(1)}, ..., x_i^{(n)}) \|^r\right)^{1/r} \\ \leq C \sup_{(\varphi_1,..,\varphi_n) \in B_{X_1^*} \times \cdots \times B_{X_n^*}} \left(\sum_{i=1}^m |\varphi_1(x_i^{(1)})...\varphi_n(x_i^{(n)})|^s\right)^{1/s}$$

for every $m \in \mathbb{N}$, $x_i^{(k)} \in X_k$ with $k = 1, ..., n$ and $i = 1, ..., m$. We denote by $\|T\|_{si(r;s)}$ the smallest number C that satisfies this inequality.

This concept of multilinear summability is due to Pellegrino et al. (2012a), and it was inspired by Çalışkan and Pellegrino (2007).

For $1 \leq s \leq r < \infty$, let us choose the following parameters in the abstract framework of Sect. 2:

$$\begin{cases} u = 1 \\ \alpha = r \text{ and } \alpha_1 = s \\ K_1 = B_{X_1^*} \times \cdots \times B_{X_n^*} \\ E_l = \{0\} \subset X_l \\ \mathcal{H} = \mathcal{L}(X_1, ..., X_n; Y) \\ R_1((\varphi_1, ..., \varphi_n), x^{(1)}, \ldots, x^{(n)}, 0, ..., 0) = \left|\varphi_1(x^{(1)})...\varphi_n(x^{(n)})\right|. \end{cases}$$

With these choices we have that an n-linear mapping $T \in \mathcal{L}(X_1, ..., X_n; Y)$ is R_1-abstract $(r; s)$ -summing precisely when T is $(r; s)$-semi-integral.

Applying Corollary 2.4, we have the splitting property for the $(r; s)$-semi-integral multilinear operators.

Corollary 3.8 *Suppose that* $1 \le s \le r < \theta, \delta < \infty$ *such that* $\frac{1}{r} = \frac{1}{\theta} + \frac{1}{\delta}$, *and let* $T \in \mathcal{L}_{si,(r;s)}(X_1, ..., X_n; Y)$. *Then, given* $m \in \mathbb{N}$ *and* $(x_i^{(k)})_{i=1}^m \subset X_k$, *with* $k = 1, ..., n$, *there are* $(\lambda_i)_{i=1}^m \subset \mathbb{K}$ *and* $(y_i)_{i=1}^m \subset Y$ *such that*

$$T\left(x_i^{(1)}, \ldots, x_i^{(n)}\right) = \lambda_i y_i \quad \text{for all } i = 1, ..., m$$

and

$$\left(\sum_{i=1}^m |\lambda_i|^\delta\right)^{\frac{1}{\delta}} \sup_{y^* \in B_{Y^*}} \left(\sum_{i=1}^m |y^*(y_i)|^\theta\right)^{\frac{1}{\theta}}$$
$$\le \|T\|_{si(r;s)} \sup_{(\varphi_1,..,\varphi_n) \in B_{X_1^*} \times \cdots \times B_{X_n^*}} \left(\sum_{i=1}^m |\varphi_1(x_i^{(1)}) ... \varphi_n(x_i^{(n)})|^s\right)^{1/s}.$$

4 Final Comments

In Sect. 3, we obtain a splitting property for the following classes of linear and nonlinear operators:

• **Linear cases:** $(r; s)$-summing, $(r; s; G)$-summing and $(\ell_s^{strong}; \ell_r)$-summing operators.

• **Nonlinear cases:** Lipschitz (r, s)-summing, absolutely $(r; r_1, \ldots, r_n)$-summing multilinear, Lipschitz $(r; s)$-summing multilinear, strongly $(r; s)$-summing multilinear and $(r; s)$-semi integral multilinear operators.

Remark 4.1 The case $s = r$ in Corollary 3.1 follows from the Maurey–Pietsch splitting theorem. However, when $s < r$, Corollary 3.1 provides a new splitting property for the absolutely $(r; s)$-summing linear operators.

Remark 4.2 The splitting property for the $(r; s; G)$-summing and $(\ell_s^{strong}; \ell_r)$-summing operators is proved in Popa (2017, Corollary 1 and 2) in the case $s = r$. When $s < r$, Corollaries 3.3 and 3.4 provide new splitting properties for these classes of operators.

Remark 4.3 The splitting property established for Lipschitz (r, s)-summing, absolutely $(r; r_1, \ldots, r_n)$-summing multilinear, Lipschitz $(r; s)$-summing multilinear, strongly $(r; s)$-summing multilinear, and $(r; s)$-semi integral multilinear operators appears to be entirely new.

Author Contributions A.L. and J.S. proved the main results. All authors provided the applications, wrote and reviewed the main manuscript text.

Funding The Article Processing Charge (APC) for the publication of this research was funded by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) (ROR identifier: 00x0ma614).

Data Availability No datasets were generated or analysed during the current study.

Declarations

Conflict of interest The authors declare no competing interests.

References

Alencar, R., Matos, M.C.: Some Classes of Multilinear Mappings Between Banach Spaces, vol. 12. Publicaciones del Departamento de Análisis Matemático, Universidad Complutense Madrid (1989)

Angulo-López, J.C., Fernández-Unzueta, M.: Lipschitz p-summing multilinear operators. J. Funct. Anal. **279**(4), 108572 (2020)

Araújo, G., García-Santiuste, A., Muñoz-Fernández, G.A.: Geometry of the space of quadratic forms on triangles and applications. Bull. Braz. Math. Soc. **56**(3), 47 (2025)

Arunachalam, S., Chakraborty, S., Koucký, M., Saurabh, N., de Wolf, R.: Improved bounds on Fourier entropy and Min-entropy. (English summary). In: 37th International Symposium on Theoretical Aspects of Computer Science, Art. pp. 45–19. LIPIcs. Leibniz Int. Proc. Inform., 154, Schloss Dagstuhl. Leibniz-Zent. Inform., Wadern (2020)

Bayart, F.: Multiple summing maps: coordinatewise summability, inclusion theorems and p-Sidon sets. J. Funct. Anal. **274**, 1129–1154 (2018)

Belacel, A., Bougoutaia, A., Macedo, R., Rueda, P.: Intermediate classes of nuclear multilinear operators. Bull. Braz. Math. Soc. **54**(4), 51 (2023)

Blasco, O., Signes, T.: Some classes of p-summing type operators. Bol. Soc. Mat. Mex. III. Ser. **9**, 119–133 (2003)

Botelho, G., Braunss, H.A., Junek, H.: Almost p-summing polynomials and multilinear mappings. Arch. Math. **76**, 109–118 (2001)

Botelho, G., Pellegrino, D., Rueda, P.: Pietsch's factorization theorem for dominated polynomials. J. Funct. Anal. **243**, 257–269 (2007)

Botelho, G., Pellegrino, D., Rueda, P.: Summability and estimates for polynomials and multilinear mappings. Indag. Math. **19**, 23–31 (2008)

Botelho, G., Maia, M., Pellegrino, D., Santos, J.: A unified factorization theorem for Lipcchitz summing operators. Q. J. Math. **70**, 1521–1533 (2019)

Çalışkan, E., Pellegrino, D.: On the multilinear generalizations of the concept of absolutely summing operators. Rocky Mt. J. Math. **37**, 1137–1154 (2007)

Chávez-Domínguez, J.A.: Duality for Lipschitz p-summing operators. J. Funct. Anal. **261**, 387–407 (2011)

Chávez-Domínguez, J.A.: Lipschitz (q, p)-mixing operators. Proc. Am. Math. Soc. **140**, 3101–3115 (2012)

Chen, D., Zheng, B.: Remarks on Lipschitz p-summing operators. Proc. Am. Math. Soc. **139**, 2891–2898 (2011)

Chen, D., Zheng, B.: Lipschitz p-integral operators and Lipschitz p-nuclear operators. Nonlinear Anal. **75**, 5270–5282 (2012)

Defant, A., Floret, K.: Tensor Norms and Operator Ideals. Math. Studies, vol. 176. Elsevier, North-Holland, Amsterdam (1993)

Diestel, J., Jarchow, H., Tonge, A.: Absolutely Summing Operators. Cambridge University Press, Cambridge (1995)

Dimant, V.: Strongly p-summing multilinear mappings. J. Math. Anal. Appl. **278**, 182–193 (2003)

Farmer, J., Johnson, W.B.: Lipschitz p-summing operators. Proc. Am. Math. Soc. **137**, 2989–2995 (2009)

Grothendieck, A.: Résumé de la théorie métrique des produits tensoriels topologiques. Bol. Soc. Mat. São Paulo **8**, 1–79 (1953)

Kislyakov, S.: Absolutely summing operators on the disk algebra. St. Petersb. Math. J. **3**, 705–774 (1992)
Lindenstrauss, J., Pełczyński, A.: Absolutely summing operators in $\mathcal{L}_p$-spaces and their applications. Studia Math. **29**, 275–326 (1968)
Macedo, R., Pellegrino, D., Santos, J.: Nonlinear variants of a theorem of Kwapień. Isr. J. Math. **247**, 217–231 (2022)
Matos, M.C.: Nonlinear absolutely summing multilinear mappings between Banach spaces. Math. Nachr. **258**, 71–89 (2003a)
Matos, M.C.: Fully absolutely summing and Hilbert–Schmidt multilinear mappings. Collectanea Math. **54**, 111–136 (2003b)
Maurey, B.: Théorémes de Factorisation Pour Les Opérateurs Linéaires à Valeurs Dans Les Espaces L_p, Astérisque, vol. 11. Société Mathématique de France, Paris (1974)
Mityagin, B., Pełczyński, A.: Nuclear operators and approximative dimensions. In: Proceedings International Congress of Mathematics, Moscow (1966)
Montanaro, A.: Some applications of hypercontractive inequalities in quantum information theory. J. Math. Phys. **53**(12), 122206 (2012)
Núñez-Alarcón, D., Serrano-Rodríguez, D.M., Teixeira, K.: Sharp exponents for anisotropic Hardy-Littlewood type of inequalities. Bull. Braz. Math. Soc. **55**(3), 37 (2024a)
Núñez-Alarcón, D., Santos, J., Serrano-Rodríguez, D.M.: Unified Grothendieck's and Kwapień's theorems for multilinear operators. Bull. Braz. Math. Soc. **55**(1), 3 (2024b)
Pellegrino, D., Santos, J.: On summability of nonlinear maps: a new approach. Math. Z. **270**, 189–196 (2012)
Pellegrino, D., Souza, M.: Fully summing multilinear and holomorphic mappings into Hilbert spaces. Math. Nachr. **278**, 877–887 (2005)
Pellegrino, D., Souza, M.: Fully and strongly almost summing multilinear mappings. Rocky Mt. J. Math. **36**, 683–698 (2006)
Pellegrino, D., Santos, J., Seoane-Sepúlveda, J.B.: Some techniques on nonlinear analysis and applications. Adv. Math. **229**, 1235–1265 (2012a)
Pellegrino, D., Santos, J., Seoane-Sepúlveda, J.B.: A general extrapolation theorem for absolutely summing operators. Bull. Lond. Math. Soc. **44**, 1292–1302 (2012b)
Pellegrino, D., Santos, J., Serrano-Rodríguez, D., Teixeira, E.: A regularity principle in sequence spaces and applications. Bull. Sci. Math. **141**, 802–837 (2017)
Pietsch, A.: Absolut p-summierende Abbildungen in normierten Räumen. Studia Math. **27**, 333–353 (1967)
Pietsch, A.: Ideals of multilinear functionals. In: Proceedings of the Second International Conference on Operator Algebras, Ideals and Their Applications in Theoretical Physics, pp. 185–199. Teubner-Texte, Leipzig (1983)
Pietsch, A.: Eigenvalues and s-Numbers, Cambridge Studies in Advance Mathematics, vol. 13. Cambridge University Press, Akad Verlagsgesellschaft Geest Portig K-G, Cambridge, Leipzig (1987)
Popa, D.: The splitting property for (p, S)-summing operators. Rev. R. Acad. Cienc. Exactas, Fís. Nat. Ser. A. Mat. **1**, 167–175 (2017)
Raposo, A., Jr., Serrano-Rodríguez, D.M.: Coefficients of multilinear forms on sequence spaces. Bull. Braz. Math. Soc. **54**(3), 43 (2023)