DOI 10.59277/RoAJ.2025.3.04

A TAKAGI-SUGENO FUZZY ALGORITHM FOR SOLAR IRRADIATION FORECASTING

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Abstract. A new model for predicting daily solar irradiation based on a Takagi-Sugeno fuzzy algorithm is reported. The study was conducted with data measured at the meteorological station of Timisoara, Romania. The model construction and its operation are presented in detail. The proposed model is based on a simple algorithm with two input variables and one output variable. The daily clearness index, calculated from the measured daily global solar irradiation represents the physical quantity processed by the fuzzy algorithm. Basically, the model forecasts the clearness index at time *t* on the basis of two previous values, measured at time *t-1* and *t-2*. An assessment of the accuracy of the algorithm was performed. Overall results show that the new forecasting model achieves an acceptable accuracy. The simplicity of the model makes it convenient for potential users. The algorithm can be applied successfully in other locations only by re-computing the model coefficients.

Key words: solar irradiation, fuzzy systems, forecasting, model accuracy.

1. INTRODUCTION

Models for forecasting solar resources represent a useful tool in operating the power grids. The forecasting procedures are commonly developed using traditional statistical methods. In the last years, artificial intelligence (AI) has become an increasingly used tool in modelling solar radiation. AI-based instruments have been introduced in forecasting solar resources too, aiming to improve the forecasts accuracy (Blaga *et al.*, 2019; Ahmed *et al.*, 2020). The fuzzy sets theory is a class of AI modeling widely used in science and engineering (Zadeh, 1965). There are many studies that deal with solar radiation modeling using fuzzy sets theory (Mellit, 2008; Boata and Paulescu, 2014; Boata and Pop, 2015; Paulescu *et al.*, 2017; Benmouiza and Cheknane, 2019). A fuzzy system consists of a map between input, named antecedent part, and output, named consequent part, represented by if - then rules. Two major classes of fuzzy systems can be defined. The first class contains the so-called fuzzy logic models. In this class, the antecedent and the consequent parts are linguistic expressions. The second class includes the Takagi-Sugeno fuzzy models (Takagi and Sugeno, 1985), where the consequent part is represented by a mathematical function and only the antecedent part is a linguistic expression. Additional references can be found in Zimmermann (1996). The solar irradiance at the Earth surface can be defined as superposition of two terms, one deterministic and the other stochastic. The deterministic variation of solar irradiance, seasonal and diurnal, is described by astronomical relationships. The stochastic component can be isolated by means of clearness index (Liu and Jordan, 1960):

$$k_{t,i} = G/G_{ext} \tag{1}$$

In Equation (1), G represents the horizontal global solar irradiance at ground level and

$$G_{ext} = G_{SC}\,\varepsilon\,\sin h \tag{2}$$

is the horizontal extraterrestrial irradiance. G_{SC} = 1366.1 W / m^2 (Gueymard, 2004) denotes the solar constant, ε is the Earth's orbit eccentricity correction factor and h is the sun elevation angle.

Clearness index, defined by Equation (1), represents a measure of the atmospheric transparency. Clearness index can also be defined for global solar irradiation over a given time interval. Tovar-Pescador (2008) presents a review of statistical behavior of solar radiation components based on clearness index.

Let's denote by H and H_{ext} the daily global horizontal solar irradiation at the ground level and the daily horizontal solar irradiation at the top of the atmosphere, respectively. Daily clearness index for global solar irradiation can be defined as:

$$k_t = H/H_{ext} \tag{3}$$

In this paper a new model based on Takagi-Sugeno fuzzy technique for forecasting daily global solar irradiation is proposed. Daily clearness index for global solar irradiation is the physical quantity processed by the algorithm. As a case study, the model development is illustrated with data measured at the radiometric station of Timisoara, Romania. The current model is based on the results obtained by us in Boata (2018). The difference lies in the use of the fuzzy clustering procedure for partitioning and defining membership functions for the main linguistic input variable.

2. MODEL DESCRIPTION AND DATABASE

The Takagi-Sugeno fuzzy algorithm was developed using 1095 data of daily global solar irradiation (H_t) recorded over three years 1998 - 2000 at the station Timisoara (latitude 45°47' N, longitude 21°17' E, elevation 80 m). Data was used for training fuzzy algorithm, *i.e.* establishing attributes and membership functions, developing the rule table and fitting the output functions. Data measured during a period of four years, 2001, 2021, 2022 and 2023 were used to test the model. Data were collected from the World Radiation Data Center, St. Petersburg, Russia (WRDC, 2024), a global repository for radiometric data. Firstly, the raw data was pre-processed aiming to compute clearness index, k_t (Equation 3). Mathematically, the database used for constructing the proposed fuzzy model is defined as:

$$M = \{(x_t, y_t)\},\ \ t = 2..1095 \tag{4}$$

where:

$$x_t = \{k_{t-1}, k_{t-2}\},\ \ y_t = \{k_t\} \tag{5}$$

The input variables k_{t-1} and k_{t-2} are the clearness index values measured at the times $t-1$ and $t-2$, respectively. The output variable k_t is clearness index forecasted by the Takagi-Sugeno algorithm for the moment t. Practically, in this study we have constructed a model for one day ahead forecasting of daily global solar irradiation. The forecasted value depends only on the two previous measured values.

In compliance with the Takagi-Sugeno fuzzy theory, a number of attributes for every input variable were assigned. Using the fuzzy clustering procedure, five attributes were established for the variable k_{t-1}, denoted $B1$, $B2$, $B3$, $B4$ and $B5$. For the variable k_{t-2} only two attributes were assigned, denoted $A1$ and $A2$. For k_{t-1}, the membership functions were generated computationally, by using the fuzzy clustering procedure (Zimmermann, 1996). For each attribute the membership function is expressed by the equation:

$$mB_i(k_{t-1}) = \begin{cases} \frac{c_{1,i}+c_{3,i}\cdot k_{t-1}+c_{5,i}\cdot k_{t-1}^2+c_{7,i}\cdot k_{t-1}^3+c_{9,i}\cdot k_{t-1}^4}{1+c_{2,i}\cdot k_{t-1}+c_{4,i}\cdot k_{t-1}^2+c_{6,i}\cdot k_{t-1}^3+c_{8,i}\cdot k_{t-1}^4} \\ if\ \frac{c_{1,i}+c_{3,i}\cdot k_{t-1}+c_{5,i}\cdot k_{t-1}^2+c_{7,i}\cdot k_{t-1}^3+c_{9,i}\cdot k_{t-1}^4}{1+c_{2,i}\cdot k_{t-1}+c_{4,i}\cdot k_{t-1}^2+c_{6,i}\cdot k_{t-1}^3+c_{8,i}\cdot k_{t-1}^4} \leqslant 1 \\ 1\ otherwise \end{cases},\ i = 1..5 \tag{6}$$

The values of the coefficients $c_{j,i}$, *i = 1..5, j = 1..9*, are listed in Table 1. Figure 1 shows the membership functions of all five attributes.

For the second input variable, k_{t-2}, the membership functions have been chosen as standard trapezoidal shape (Figure 2):

$$mA_1(k_{t-2}) = \begin{cases} max\left(0, 1-\frac{k_{t-2}-d_1}{e_1-d_1}\right)\ if\ d_1 < k_{t-2} \\ 1\ otherwise \end{cases} \tag{7a}$$

$$mA_2(k_{t-2}) = \begin{cases} max\left(0, \frac{k_{t-2}-d_2}{e_2-d_2}\right)\ if\ k_{t-2} < e_2 \\ 1\ otherwise \end{cases} \tag{7b}$$

Table 1

The coefficients of the variable k_{t-1} membership functions

$c_{i,j}$		i = 1	2	3	4	5
j	1	0.583	0.235	0.056	0.104	0.042
	2	-13.671	-14.358	-9.762	0.190	-9.270
	3	-6.133	-3.985	-0.588	-0.861	-0.499
	4	73.298	82.970	35.759	15.624	30.995
	5	23.632	22.962	2.248	0.900	2.219
	6	-181.685	-219.166	-58.219	-90.382	-44.211
	7	-39.435	-51.764	-3.704	4.390	-4.419
	8	177.403	220.392	35.542	101.792	23.031
	9	24.073	39.588	2.225	-6.217	3.347

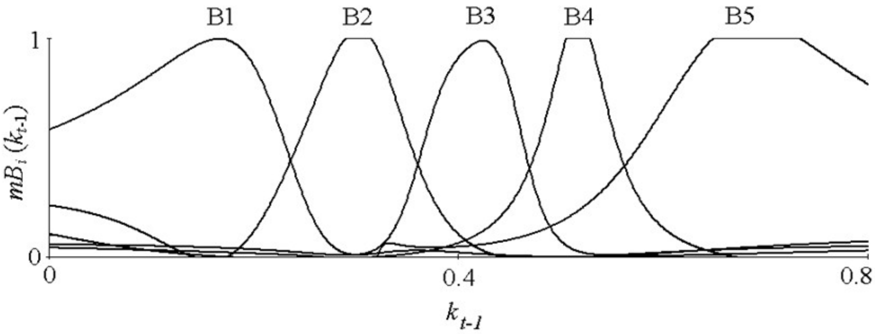

Fig. 1 – Membership functions associated to the input variable k_{t-1}.

The coefficients in Equations (7a) – (7b) are: $d_1 = d_2 = 0.2$ and $e_1 = e_2 = 0.6$.

The rule-base represents a mapping of the inputs to the output of the fuzzy system. This was elaborated in a heuristic way using the training data set and following the Takagi-Sugeno structure of a fuzzy model. In this model, the rule-base consists of a collection of 10 rules, expressed as:

$$\#R_k : if\ k_{t-2}\ is\ Ak\ and\ if\ k_{t-1}\ is\ Bk\ then\ y_k = y_k(k_{t-1}, k_{t-2}),\ k = 1..10 \quad (8)$$

where Ak and Bk are fuzzy sets corresponding to the partitioned domains of the input variables k_{t-2} and k_{t-1}. The matrix of the rule-base is listed in Table 2.

The output functions y_i were chosen in the simplest form, *i.e.* linearly in k_{t-1} and k_{t-2}:

$$y_i(k_{t-1}, k_{t-2}) = a_i + b_i k_{t-1} + c_i k_{t-2},\ i = 1..10 \quad (9)$$

The coefficients of the output functions y_i were estimated using the bivariate least square regression. The coefficients are listed in Table 3.

This section presented in detail the procedure of development of the new Takagi-

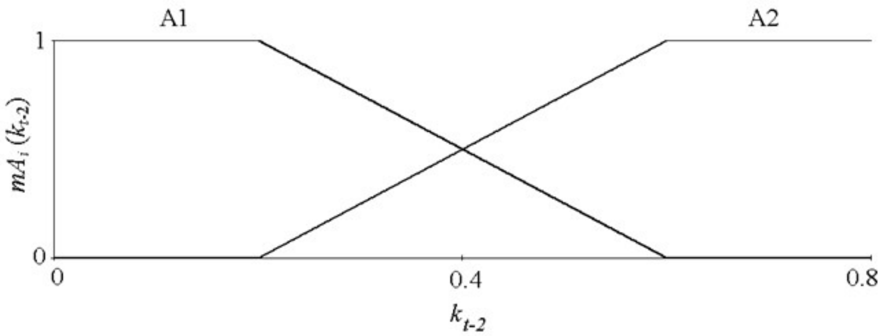

Fig. 2 – Membership functions associated to the input variable k_{t-2}.

Table 2

The system rules-base matrix. Each rule is a fuzzy implication in the Equation (8) sense

The rules-base		k_{t-1}				
		B1	B2	B3	B4	B5
k_{t-2}	A1	y_1	y_2	y_3	y_4	y_5
	A2	y_6	y_7	y_8	y_9	y_{10}

Sugeno fuzzy algorithm. At this stage, the algorithm is fully operational, for every input pair k_{t-1} and k_{t-2} the algorithm returns the output value k_t.

Table 3

Coefficients in the Equation (9)

i	1	2	3	4	5	6	7	8	9	10
a_i	0.220	0.411	0.609	-0.289	-0.272	-0.036	-0.002	0.373	0.273	0.273
b_i	0.106	0.011	0.102	-0.001	0.564	0.362	0.240	0.405	0.045	0.045
c_i	0.285	-0.090	-0.501	1.427	0.976	1.221	0.937	-0.406	0.423	0.423

3. MODEL OPERATION

This section illustrates step by step the Takagi-Sugeno fuzzy algorithm operation. Three values of daily global solar irradiation measured on 20, 21 and 22 August 2001, corresponding to the Julian days 232, 233 and 234, are used for this illustration. From these data, the following values of the clearness index were calculated: $k_{232} = 0.657$, $k_{233} = 0.621$ and $k_{234} = 0.639$. The first step in the operation of a fuzzy algorithm is fuzzification. At this step, the confidence levels of the attributes for the input variable are calculated with Equation (6) and Equations (7a) – (7b):

$$
\begin{gathered}
mB1(k_{233}) = 0.006 \\
mB2(k_{233}) = 0.024 \\
mB3(k_{233}) = 0.025 \\
mB4(k_{233}) = 0.069 \\
mB5(k_{233}) = 0.831 \\
mA1(k_{232}) = 0 \\
mA2(k_{232}) = 1
\end{gathered}
$$

The second step in the operation of a fuzzy algorithm is the inference. According to the rule-base, all the rules are set up. The fuzzy operator AND is applied to generate the fuzzy output values:

$$
\begin{gathered}
m_1 = min[mA1(k_{232}), mB1(k_{233})] = 0 \\
m_2 = min[mA1(k_{232}), mB2(k_{233})] = 0 \\
m_3 = min[mA1(k_{232}), mB3(k_{233})] = 0 \\
m_4 = min[mA1(k_{232}), mB4(k_{233})] = 0 \\
m_5 = min[mA1(k_{232}), mB5(k_{233})] = 0 \\
m_6 = min[mA2(k_{232}), mB1(k_{233})] = 0.006 \\
m_7 = min[mA2(k_{232}), mB2(k_{233}] = 0.024 \\
m_8 = min[mA2(k_{232}), mB3(k_{233})] = 0.025 \\
m_9 = min[mA2(k_{232}), mB4(k_{233})] = 0.069 \\
m_{10} = min[mA2(k_{232}), mB5(k_{233})] = 0.831
\end{gathered}
$$

Finally, in the last step of the algorithm, namely defuzzification, the forecast (a crisp value) is issued (Takagi and Sugeno, 1985):

$$
k_{234,est} = \sum_{i=1}^{10} m_i y_i \Bigg/ \sum_{i=1}^{10} m_i = 0.581 \tag{10}
$$

Therefore, the forecasted value of the clearness index in the Julian day 234 is $k_{234,est} = 0.581$, with a relative error $[(k_{234,est} - k_{234})/k_{234}] \cdot 100 = -9.07\%$.

4. ASSESSMENT OF MODEL PERFORMANCE

Data recorded in four different years 2001, 2021, 2022 and 2023 at the station of Timisoara were used to assess the model performance. Figure 3 shows the time dependence of measured and forecasted daily clearness index. A good agreement between measured and forecasted values can be observed.

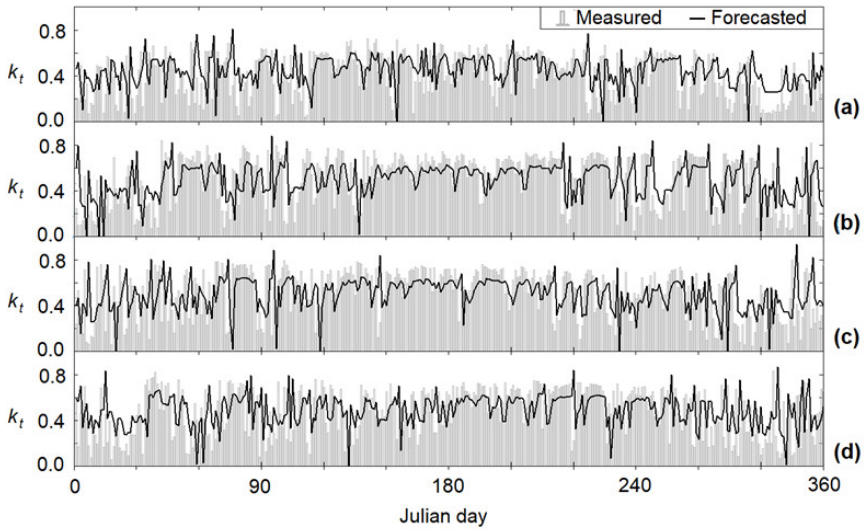

Fig. 3 – Comparison between forecasted and measured daily clearness index at the station of Timisoara, in four different years: (a) 2001, (b) 2021, (c) 2022 and (d) 2023.

The model accuracy was measured in term of two statistical indicators: the root mean square error $(rmse)$:

$$rmse = \left[\frac{1}{N} \cdot \sum_{i=1}^{N} (F_i - y_i)^2\right]^{1/2} \tag{11}$$

and the mean bias error (mbe):

$$mbe = \frac{1}{N} \cdot \sum_{i=1}^{N} (F_i - y_i) \tag{12}$$

where y_i and F_i are i-th measured and computed values, respectively, N denotes the number of measurements taken into account. The statistical indicators of accuracy of the proposed model are gathered in Table 4.

The final aim of the proposed Takagi-Sugeno fuzzy algorithm is the forecasting of daily global solar irradiation. Using Equation (3), the time series of forecasted daily global solar irradiation can be reconstructed from the time series of clearness

Table 4

Statistical indicators of accuracy when the proposed forecasting model was tested against measured data

Forecasted quantity	Statistical indicator	2001	2021	2022	2023
Daily clearness index	$rmse$	0.207	0.220	0.221	0.219
	mbe	0.030	-0.001	-0.014	-0.001
Daily global solar irradiation	$rmse$ [kWh/m2]	1.587	1.546	1.534	1.593
	mbe [kWh/m2]	0.094	-0.142	-0.223	-0.109

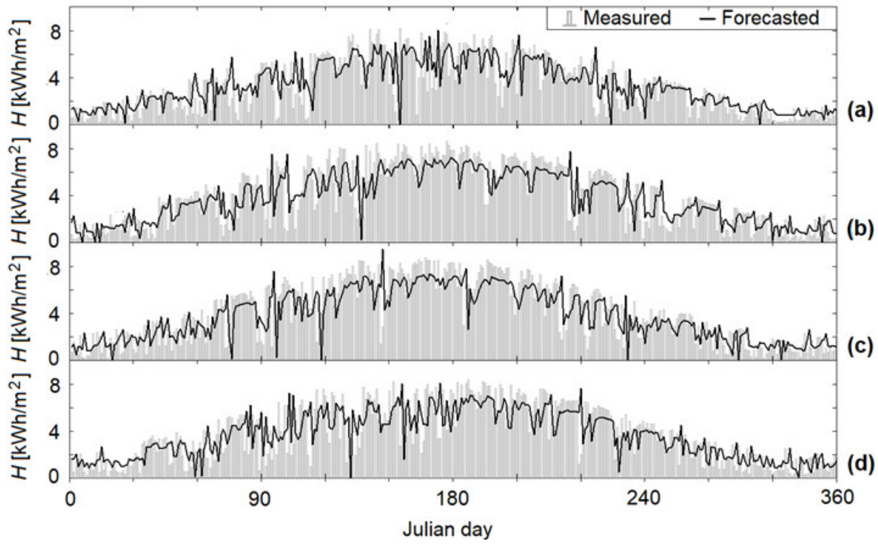

Fig. 4 – Comparison between forecasted and measured daily global solar irradiation at the station of Timisoara in four different years: (a) 2001, (b) 2021, (c) 2022 and (d) 2023.

index forecasts. Figure 4 displays the results. The values of the statistical indicators computed with Equation (11) and Equation (12) are also listed in Table 4, indicating that the forecasting accuracy is relatively constant over time.

5. CONCLUSIONS

A new Takagi-Sugeno fuzzy model for forecasting daily global solar irradiation is proposed. The variable directly processed by the fuzzy algorithm is clearness index computed from measured global solar irradiation. Based on the results it can be concluded that the accuracy of the proposed model is acceptable for practice. The balance between simplicity and accuracy of the fuzzy model developed in this study recommends it for photovoltaics applications. For potential users the model construction is described step by step. The results presented for Timisoara, the radiometric station from which data were collected for the development and testing of the model, can be viewed as a case study. The model can be applied easily in any other location, only a recalculation of the membership functions coefficients being necessary.

REFERENCES

Ahmed, R., Sreeram, V., Mishra, Y. and Arif, M.D.: 2020, *Renewable and Sustainable Energy Reviews*, **1**24, 109792.

Benmouiza, K. and Cheknane, A.: 2019, *Theor Appl Climatol*, **1**37, 31.

Blaga, R., Sabadus, A., Stefu, N., Dughir, C., Paulescu, M. and Badescu, V.: 2019, *Progress in Energy and Combustion Science*, **7**0, 119-.

Boata, R.: 2018, *Annals of West University of Timisoara - Physics, **60** (1), 38.*

*Boata, R., Paulescu, M.: 2014, Environmental Engineering and Management Journal, **1**3(12), 3045.*

*Boata, R. and Pop, N.: 2015, Romanian Journal of Physics, **6**0 (3-4), 593.*

*Gueymard, C.A.: 2004, Solar Energy, **7**6, 423.*

*Liu, B.Y. and Jordan, R.C.: 1960, Solar Energy, **4**, 1.*

*Mellit, A.: 2008, Int. J. Artif. Intell. Soft Comput., **1**(1), 52.*

*Paulescu, M., Brabec, M., Boata, R. and Badescu V.: 2017, Energy, **1**21, 792.*

*Takagi, T. and Sugeno, M.: 1985, EEE Trans. Syst. Man Cybern., **1**5, 116-.*

Tovar-Pescador, J.: 2008, In: Badescu, V. (Ed.), In Modeling Solar Radiation at the Earth's Surface. Springer, Berlin, 55.

*Zadeh, L.A.: 1965, Inf. Control, **8**, 338.*

Zimmermann, H.J.: 1996, Third ed. Kluwer Academic Publishers, Boston.

World Radiation Data Center,WRDC.:2024, St. Petersburg, Russia. Available online at: http://wrdc.mgo.rssi.ru/.

Received on 12 May 2025