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The most cost-effective way to find the best cosmetic doctor, a mathematical proof

Cheuk Hung Lee, MBBS (HK), FHKAM (MED), FHKCP, MScPD (Cardiff), MRCP (UK), DPD (Wales), DipDerm (Glasgow), PGDipClinDerm (London), MRCP (London), GradDipDerm (NUS), DipMed (CUHK), Kar Wai Alvin Lee, MBChB (CUHK), DCH (Sydney), Dip Derm (Glasgow), MScClinDerm (Cardiff), MScPD (Cardiff), DipMed (CUHK), DCH (Sydney), Kwin Wah Lisa Chan, MBChB (CUHK), MScPD (Cardiff), PgDipPD (Cardiff), PGDipClinDerm (Lond), DipMed (CUHK), DCH (Sydney)

Ever Keen Medical Centre, Hong Kong

Background: In the pursuit of optimal healthcare, patients often face significant challenges in selecting suitable cosmetic physicians, leading to a phenomenon known as "doctor shopping." This process is associated with considerable financial, temporal, and emotional costs.

Objective: Given the importance of informed decision-making in enhancing healthcare quality, our study introduces a mathematical model grounded in optimal stopping theory to streamline the choice of cosmetic doctors or any healthcare providers.

Methods: We induce a mathematical model derives from the formulation we described in the section mathematical model and result, where we can derive from the given condition that there existed the maximum probability of finding the best cosmetic doctor.

Results: The best choice is to reject the first 36.8% of available visited cosmetic doctors and chooses the first candidate who surpasses all previously consulted candidates.

Conclusion: The mathematical model and optimal stopping strategy present a groundbreaking approach to the long-standing problem of doctor shopping. By equipping patients with a systematic methodology for selecting healthcare providers, this model acknowledges the complexities involved in healthcare decision-making while offering a pathway to more informed and effective choices. Furthermore, we also believe that this method also applicable to find best doctor in other specialties suitable to that particular patient instead of finding his/her best cosmetic doctor only.

Keywords: cost-effectiveness analysis; decision making; health personnel; mathematics; problem solving; statistics

Introduction

Patients often "doctor shop" to find the best cosmetic doctor to manage their medical condition. The ideal approach might seem to be consulting every available cosmetic doctor. However, this is associated with significant opportunity costs, not only the financial cost but also the time cost, and may defer the treatments; even though the patient finds the best doctor, deferring the management may not be the best solution to the patients' medical problem [1-3].

Doctor shopping involves visiting multiple healthcare providers to obtain prescriptions for controlled substances. It is often associated with substance abuse, addiction, and fraudulent behavior. Cosmetic patients who engage in doctor shopping

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Corresponding author: Kar Wai Alvin Lee
E-mail: alvin429@yahoo.com

often present with symptoms such as chronic pain, anxiety, or depression, and may seek prescription medications such as opioids, benzodiazepines, or stimulants. This is a growing public health concern in many countries worldwide [4-6].

Detecting and preventing doctor shopping poses a challenge. Healthcare professionals often struggle to distinguish between patients genuinely seeking medical treatment and those engaged in fraudulent practices. While some individuals may doctor shop for legitimate reasons, others do so to obtain drugs for non-medical purposes. This behavior can result in adverse side effects, drug interactions, and mortality.

Prescription medication abuse is an indication for doctor shopping. Individuals who use prescription medications for non-medical purposes may attempt to obtain more drugs than their legal allotment. Chronic pain is another key factor; patients with chronic pain may feel that they are not receiving adequate treatment from their healthcare providers and may seek additional prescriptions elsewhere. Moreover, patients may doctor shop due to a lack of access to healthcare or insurance coverage or due to stigmatization associated with certain medical conditions [7-9].

Overall, doctor shopping is a critical issue requiring the attention of healthcare providers, law enforcement providers, and policymakers. Therefore, there is a need for improved education, preventive strategies, and comprehensive treatment options for substance abuse and addiction. Addressing the underlying causes of doctor shopping is vital for reducing the harm it poses [10-12].

Thus, identifying a cost-effective way to find the best cosmetic doctor becomes an important issue, and the best way is to spend the least amount of money and time to find the best cosmetic doctor available in the market. The most common method employed is word-of-mouth referral, wherein patients rely on recommendations from friends or relatives who have visited certain doctors. However, such recommendations are often biased; a doctor who was ideal for a friend or family member may not be suitable for another patient, as each cosmetic patient has a different agenda and medical conditions [13-16].

There are two extreme situations in this context. The first is that the patient can visit the first cosmetic clinic, and furthermore, he/she has the best doctor among the whole list. This situation incurs the lowest possible cost, but the possibility of obtaining the best cosmetic doctor on the first attempt is low. The second extreme situation is that the patient visits all doctors on the list and finally obtains the best doctor, which is extremely costly and time-consuming. Although the patient has the best doctor, it is certainly not cost-effective.

We can maximize profits, costs, and time simultaneously. Therefore, in many optimization problems, these tasks can be combined and appear within the same problem framework, which is defined as an objective function [16-20].

There are two major categories of optimization problems: continuous optimization and discrete optimization.

Regarding constraint conditions, the ideal is beautiful, but reality is tough. In real life, we may encounter various problems, such as limited budgets, restricted timeframes, and external mandatory conditions. Similar to the objective function, these constraint conditions do not exist alone, and multiple constraint conditions may exist for the same problem. For a specific optimization problem, the more complex the constraint conditions, the more difficult it is to solve [21,22].

Based on this, optimization problems can be further classified into two categories based on their constraint conditions and variable types: continuous optimization [23,24] and discrete optimization [25,26].

Continuous optimization does not break in the line, while the variable values in discrete optimization are discontinuous records. Discrete optimization is more difficult to solve because it has an additional constraint condition—a set that is not continuous. Many times, we need our variables to be integers or come from a given interval; therefore, discrete optimization is more difficult to solve than continuous optimization, and the two algorithms will be quite different.

From an academic perspective, continuous and discrete optimizations correspond to two independent disciplines. Discrete optimization may be applied more frequently in statistical and big data-related scenarios.

Starting from the objective function, its optimal values are divided into two types: local optimum [27] and global optimum [28]. The local optimum is the lowest in the local area, and we call this the local optimum value. However, when the entire picture is considered, there may exist a point where it is the lowest; therefore, we call that point the global optimum value.

Determining a local optimum value is relatively easy because it is necessary to look at a small amount of nearby information to accurately determine whether it is a local optimum. In practical applications, knowing only the local optimum value is sufficient to solve several problems. The more difficult problem lies in the global optimum value, because the entire picture needs to be viewed as a prerequisite.

Materials and methods

The optimal stopping theory, as applied in this study, acknowledges the challenge of making irrevocable decisions—a common scenario for patients consulting cosmetic doctors sequentially. The decision of when to stop searching for a better option is guided by the statistical probability of finding the best provider based on previously gathered information. By systematically rejecting an initial group of doctors and then selecting from the remaining pool, patients can increase their chances of making a successful choice. This strategic rejection is not arbitrary; rather, it is designed to gather sufficient information to estimate the potential quality of the remaining candidates. It acknowledges the uncertainty inherent in healthcare decisions while offering a structured framework for effectively navigating that uncertainty.

Knowledge is a powerful health care tool. The proposed model enables patients to obtain essential information about each practitioner without excessive consultations. This optimization strategy is aligned with modern healthcare trends that emphasize patient engagement and informed decision-making. By understanding their evaluative process, patients can become active participants in their healthcare journey rather than passive recipients of care. Furthermore, this framework encourages patients to approach healthcare choices quantitatively, empowering them to move beyond anecdotal evidence and make decisions based on their personal preferences and medical needs. This shift has the potential to enhance patient satisfaction and improve health outcomes.

Results

Mathematical model and result

Consider a patient who wants to visit the best cosmetic doctor out of the number of doctors in the list he/she has. Doctors are visited individually in a random sequence. A decision regarding each doctor visited must be made immediately following the consultation. Once determined, they were not the best choice, and the doctor could not be recalled. During the consultation, the patient gains sufficient information to rank the doctor among all doctors visited thus far but remains unaware of the quality of the doctors yet to be consulted. This question concerns the optimal strategy for maximizing the probability of selecting the best candidate. If the decision can be deferred to the end, it can be solved using a simple maximum selection algorithm for tracking the running maximum and selecting the overall maximum at the end. The difficulty is that decisions must be made immediately, which implies cost-effectiveness.

The optimal stopping rule prescribes always rejecting the first *n/e* applicants (where e is the Euler number) who are interviewed and then stopping at the first doctor who is better than every doctor visited so far (or continuing to the last doctor if this never occurs). This stopping rule is simple and can help patients select the single best candidate approximately 36.8% of the time, regardless of whether there are 100 or 100 million doctors.

Before proceeding with the calculation, several definitions must be established:

There is a single position to be filled.

There are *n* doctors on the list that the patient is going to consult, and the value of *n* is known.

- Doctors can be unambiguously ranked from worst to best if they are all seen together.

Doctors are consulted sequentially in random order, with each order being equally likely.

- Immediately after the consultation, the visited doctor is accepted as the best fit for the patient, and the decision is irrevocable.

The decision to accept or reject a doctor as the best doctor can be based only on the relative rankings of the doctors consulted so far.

The objective was to obtain the highest probability of selecting the best doctor from the entire doctor list. This is the same as maximizing the expected payoff, with the payoff defined as one for the best applicant and zero otherwise.

These criteria were based on the “Who solved the secretary problem?” paper [29].

We assume there are n doctors, and we need to give up k doctors; according to this strategy, the probability of choosing the best doctor is P(k).

$$\begin{aligned} P(k) &= P(k+1 \text{ is the best doctor}) + P(k+2 \text{ is the best doctor}) \\ &\quad + P(k+3 \text{ is the best doctor}) + \ldots + P(n \text{ is the best doctor}) \\ &= \frac{1}{n} + \frac{1}{n}\cdot\frac{k}{k+1} + \frac{1}{n}\cdot\frac{k}{k+2} + \frac{1}{n}\cdot\frac{k}{k+3} + \ldots + \frac{1}{n}\cdot\frac{k}{n-2} + \frac{1}{n}\cdot\frac{k}{n-1} \\ &= \frac{k}{n}\left(\frac{1}{k} + \frac{1}{k+1} + \frac{1}{k+2} + \frac{1}{k+3} + \ldots + \frac{1}{n-2} + \frac{1}{n-1}\right) \\ &= \frac{k}{n}\sum_{i=k}^{n-1}\frac{1}{i} \end{aligned}$$

In order to let $x = \frac{k}{n}$,

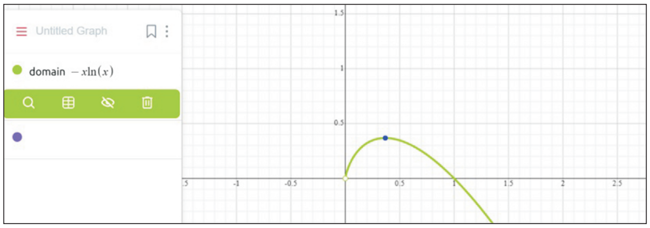

Fig. 1. Graphical demonstration of y = –x ln x.

$$P(k) = \frac{k}{n}\sum_{i=k}^{n-1}\frac{1}{i} \qquad x\int_x^1 \frac{1}{x}dt = -x\ln x$$

To maximize P(k), P’(k)=–(1+ln x)=0, we put $x=\frac{1}{e}$, where *e* is the Euler number

Now $P(k)=-\frac{1}{e}\,\mathrm{In}\,\frac{1}{e}=\frac{1}{e}$, which is approximately 36.8%

This is a very simplified mathematical proof of this situation; we already know that the answer would be $\frac{1}{e}$, and just plugged it into the answer to reach the conclusion (Fig. 1). The original mathematical proof of this optimal stopping strategy was provided by Bruss [30], but it may be too difficult for medical physicians, and we do not want to copy and paste the entire proof in this article; readers who are interested in the original proof can check the original article.

Practical illustration

If I look for the best cardiologist, I can look at the list and discover that there are 319 certified cardiologists available. The best stopping method required me to drop the first 319×36.8%=117 cardiologists before selecting the next one, who was superior to the 117 consulted cardiologists.

If I look for a palliative medicine specialist, I can look at the list and discover that there are 40 certified palliative medicine specialists. The best stopping method required me to drop the first 40×36.8%=15 palliative medicine specialists before selecting the next one, who was superior to these 15 consulted palliative medicine specialists (Fig. 2) [31].

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Statistics on fellows and trainees in all specialties

Specialty	Number of Fellows	Number of Trainees
Internal Medicine	**1,718**	**331**
Cardiology	319	46
Clinical Pharmacology & Therapeutics	9	2
Clinical Toxicology	5	0
Critical Care Medicine	120	17
Dermatology & Venereology	122	19
Endocrinology, Diabetes & Metabolism	137	25
Gastroenterology & Hepatology	241	30
Geriatric Medicine	223	27
Haematology & Haematology Oncology	92	10
Immunology & Allergy	8	0
Infectious Disease	53	9
Medical Oncology	56	11
Nephrology	167	15
Neurology	158	26
Palliative Medicine	40	9
Rehabilitation	66	9
Respiratory Medicine	226	20
Rheumatology	104	16

Updated as of 16 May 2023

Fig. 2. Statistics on fellows and trainees in all specialties of internal medicine in Hong Kong in the year 2023. Reused from Hong Kong College of Physicians. https://www.hkcp.org/docs/Synapse/synapse202308.pdf [31].

Discussion

To the best of our knowledge, this is the first study to apply a mathematical formula instead of the statistical methods typically used in medicine to solve medical problems. We can do well in cross-specialty domain knowledge, as human mind limitations make each specialty unable to really have an understanding of what another specialty is doing and what their new development is in the field. The optimal stopping theory, proven for over 40 years, is applied here for the first time in solving a real-life medical problem.

In the increasingly complex healthcare environment, selecting the most appropriate doctor remains a critical concern for patients with various medical conditions. The mathematical model and optimal stopping strategy discussed herein present a significant step toward addressing the challenges patients face in finding the most suitable healthcare provider while minimizing the associated costs. The model not only enhances decision-making efficiency but also serves as a guide for patients to navigate the complexities of doctor shopping.

In an era of escalating costs, it is imperative that patients make informed decisions that are both efficient and effective. The traditional approach in which patients consult multiple doctors in search of the best match can lead to financial strain and delays in treatment. Given that time is often a critical factor in healthcare, particularly for patients with chronic conditions, the ability to streamline the process of finding the right physician is more important than ever. The implications of the proposed mathematical model extend beyond mere cost savings and encompass the urgency and necessity of timely interventions in healthcare.

As discussed in the introduction, many patients continue to rely on word-of-mouth recommendations when choosing a physician. Although these personal experiences can provide insight, they are inherently biased and do not account for each patient's unique medical needs and preferences. The proposed model mitigates the impact of subjective biases by relying on a systematic approach to evaluate doctors and maintaining objectivity throughout the decision-making process. This framework emphasizes the need for personalized care tailored to individual patient circumstances, rather than relying on generalized recommendations.

Challenges of implementation

Although the optimal stopping strategy offers a theoretical framework for decision-making, several practical challenges may arise during its implementation. Patients often do not have the luxury of time, and delays in acute situations can lead to significant health risks. Additionally, emotional factors associated with healthcare decisions, such as trust, anxiety, and urgency, may complicate the rational calculus suggested by the model.

Healthcare providers must recognize the limitations of this model. Not all patients possess the capacity to engage with the process or interpret information in a similar manner. Therefore, it is essential for physicians and healthcare systems to support patients in understanding and applying this idea while remaining sensitive to their emotional and psychological needs.

The implications for policy and healthcare design

This model has broad implications for healthcare policy and system design. As the healthcare landscape continues to evolve, there is a growing need for systems that facilitate better decision-making among patients. This model can serve as the foundation for developing rules and recommendations concerning patient consultations, emphasizing a structured approach that reduces unnecessary visits while maximizing the likelihood of finding the most suitable doctor.

Healthcare systems can also deploy decision-support tools and digital platforms that embody these principles, educate patients about their options, and provide frameworks for improved decision-making. Such enhancements can promote cost-effective care and contribute to better patient outcomes through the systematic use of evidence-based practice.

Limitations of this study

Generalizability: These findings of this study may not be universally applicable across all medical specialties and patient demographics, necessitating further validation in diverse contexts.

Modal assumptions: The proposed model relies on assumptions about patient behavior and decision-making that may not reflect real-world complexities such as emotional factors or urgency in healthcare decisions.

Implementation challenges: Practical challenges in applying the model in clinical settings, including the patient's capacity to engage with the process and potential delays in care, may hinder its effective use.

Lack of empirical validation: The effectiveness of a model requires empirical studies to confirm its applicability and to refine its methodology in real-world scenarios.

Future research directions

Future research should investigate the applicability of this model to diverse healthcare contexts and patient populations. Investigating its adaptability to various medical specialties and demographic groups may provide valuable insights into how decision-making processes vary with patient needs and conditions. Additionally, empirical studies investigating the real-world effectiveness of the proposed model can significantly contribute to refining its applications.

Exploring the integration of digital health tools into this framework may offer further innovations in how patients navigate their healthcare journeys. The potential of artificial intelligence and machine learning to support decision-making processes, coupled with optimal stopping strategies, presents exciting avenues for future investigations that can enhance both patient experiences and outcomes.

In conclusions, the proposed mathematical model and optimal stopping strategy present a groundbreaking approach to the persistent issue of doctor shopping. By equipping patients with a systematic methodology for selecting healthcare providers, this model addresses the inherent complexities involved in healthcare decision-making while offering a pathway to more informed and effective choices. As healthcare continues to evolve, incorporating evidence-based approaches can lead to significant advancements in how patients engage with their care, ultimately embedding a sense of agency and empowerment into patient experience.

The need for cost-effective, timely healthcare choices has never been more pressing, and as recognized in this discussion, the mathematical and strategic principles introduced herein may pave the way for a more balanced, patient-centric healthcare environment.

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Conflicts of interest

The authors have nothing to disclose.

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