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Quality Innovation with Variable Demand Elasticity

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Abstract
I study product-quality innovation under monopolistic competition with variable demand elasticity preferences and general cost functions. I characterize the free-entry equilibrium and the effects of market size, marginal production cost, and fixed cost on equilibrium product quality and markups. Then I show how these results change with a small number of firms that engage in competitive interaction *a-la* Cournot. Overall, for the commonly assumed demand and cost structures, an increase in market size or marginal production cost works to decrease markups and increase product quality, while markups increase along with product quality due to an increase in fixed (entry) cost.

Keywords Quality Innovation · Variable Demand Elasticity · Monopolistic Competition · Cournot Competition

JEL Classifications L-1 · O-30

1 Introduction

The study of product-quality innovation has been central to both theoretical and empirical research on industrial organization, economic growth, and international trade. Large parts of this literature rely on the monopolistic competition framework that was proposed by Dixit and Stiglitz (1977), with constant elasticity of substitution (CES) and constant demand elasticity (CDE).[1] A recent growing research body

[1] Hereafter, I will refer to the CES specification as "CDE".

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has been studying the implications of more general preferences specifications, with Variable Demand Elasticity (VDE), to market outcomes.

A key subject that has been addressed in these analyses is the effect of market size on competitive pressure measures, such as markups and market concentration, which are neutralized under the CDE specification.[2] These issues are central to the contemporary study of the apparent increase in market concentration and market power across American industries, and to the study of the effects of globalization on market outcomes.[3]

The present study contributes to this literature an analysis of product quality innovation with the VDE additive preferences specification that was studied by Zhelobodko et al. (2012), for which demand elasticity may be decreasing or increasing with consumption level. The preference for product quality is introduced into the per-variety utility function as a factor over the utilized quantity, which is the standard in Schumpeterian growth models.[4] Product quality innovation is highly relevant to the analysis of market concentration and changing markups and their welfare implications, and particularly the relation between equilibrium markups and product quality improvement, which are explored here: Changes in quality-adjusted markups may be a more appropriate measure for changes in market power and consumers surplus.[5]

In a single-sector general equilibrium, I first characterize the market-size, marginal cost, and fixed-cost effects on equilibrium markups and product quality, under monopolistic competition: I show that the effects of market size and marginal cost on equilibrium product quality depend on whether demand elasticity decreases or increases with consumption level, and their effects on equilibrium markups depend on the firms' cost structure: the shape of the costs elasticity functions. The fixed-cost effect on equilibrium product quality is definitely positive, and the effect on equilibrium markups depends on demand type.

These results align with those that were obtained by Bykadorov and Kokovin (2017) for cost-reducing innovation in the same framework that is employed here, and with the analysis of Vives (2008) that studies cost-reducing innovation in an oligopolistic market focusing on decreasing demand elasticity.[6] These studies assume a constant marginal cost of production that can be reduced through innovation cost. The present study adds to the previous results by considering a general cost function, and by addressing the effect of changes in the marginal cost on product quality and markups. Furthermore, it considers also the possibility that the marginal cost of production increases with product quality: Higher product quality is more costly to produce.

[2] See for example (Zhelobodko et al., 2012; Bertoletti and Etro, 2016, 2017, 2021); and Matsuyama and Ushchev (2022).

[3] See for example (Shapiro and Yurukoglu, 2026; Peltzman, 2022; Basu, 2019; Berry et al., 2019; Shapiro, 2019; Bertoletti et al., 2018); and Bertoletti and Epifani (2014).

[4] As in the Grossman and Helpman (1991) canonical Schumpeterian ("Quality Ladders") growth model.

[5] See Peltzman (2022) and Berry et al. (2019).

[6] Cost-reducing innovation has been considered also as "process innovation" and "productivity innovation" in the literature; see for example (Vives, 2008) and Ganapati (2021), respectively.

Next, I show how the equilibrium outcomes and their comparative statics change when a small number of firms operate and engage in competitive interaction *a-la* Cournot, thereby also nesting the results that were obtained by Bykadorov and Kokovin (2017) and Vives (2008) within the same unified framework.

Overall, for the commonly assumed decreasing demand elasticity and increasing cost elasticities,[7] an increase in market size or marginal production cost works to decrease markups and increase product quality, while markups increase along with product quality due to an increase in fixed (entry) cost.

The remainder of the paper is organized as follows: Section 2 presents the model market. Section 3 characterizes the free-entry equilibrium and its comparative statics under monopolistic competition. Section 4 repeats the equilibrium analysis for Cournot competition, and Section 5 concludes this study.

2 The Model

I study a single-sector general equilibrium in a market that is populated with L identical agents, who are both workers and consumers. Market population L corresponds to the market size. Each worker supplies one labor unit. The wage is normalized to one and is the workers' sole income. Workers spend their income on a continuum of (horizontally) differentiated consumption goods –"varieties"– that are indexed $i \in [0, m]$. Each product variety is produced and sold by a single (different) producer, under monopolistic competition.

2.1 Preferences and Consumer Optimization

The consumer's preferences are represented by the VDE utility function studied in Zhelobodko et al. (2012):

$$U = \int_0^m u(c_i)\, di\,. \tag{1}$$

The per-variety consumption utility –"subutility"– function $u(\bullet)$ is concave and thrice differentiable. The consumption stream that is derived from each variety – c_i – is given by $c_i = q_i \cdot x_i$, where x_i and q_i designate the utilized quantity and quality, respectively. Standard consumer utility maximization, subject to the unit labor income and the given product prices, yields the per-variety inverse demand function:

$$p_i = \frac{q_i \cdot u'(q_i \cdot x_i)}{\lambda} \tag{2}$$

[7] "...Typically an increase in the number of competitors for a given total market size will decrease the residual demand for the firm and will increase the demand elasticity" - Vives (2008, p. 422).

where λ is the Lagrange multiplier: $L = U + \lambda \left[1 - \int_0^m p_i x_i\right]$. Substituting (2) into the budget constraint yields $\lambda = \int_0^m q_i \cdot x_i \cdot u'(q_i \cdot x_i)$, which is taken as given by each producer, as *m* is assumed to be sufficiently large. From (2), I also derive the product-variety demand elasticity and its absolute value, denoted $\varepsilon_{x,p}$ and $s_i(c_i)$, respectively:[8,9]

$$s_i(c_i) \equiv |\varepsilon_{c,p}| = -\frac{u'(c_i)}{c_i \cdot u''(c_i)}\,. \tag{3}$$

I will use the abbreviations DDE and IDE for decreasing and increasing demand elasticity – in absolute value – respectively. For equal consumption levels from all products, equation (3) also defines the elasticity of substitution across varieties.[10] The monopolistic competition under study is viable if demand elasticity (absolute value) is finite and greater than one: $s_i \in (1, \infty)$.

2.2 Technologies

Labor is the sole input for production and for quality innovation. The firm's output – $y \equiv x \cdot L$ – is subject to a fixed cost, *F*, and a weakly convex variable cost function, *vc*(*y*), with $vc'(y), vc''(y) \geqslant 0$, and $vc(0) = 0$. Therefore, the total cost of production is $tc(y) = F + vc(y)$. Product-quality enhancement is subject to a convex innovation (R&D) cost function, *f*(*q*), with $f'(q), f''(q) > 0$. The sum of production and innovation costs is denoted $C = tc(y) + f(q)$.

3 Monopolistic Competition

Given the technologies and preferences that were presented above, each firm out of a *m*-size mass maximizes the following profit function by choosing its output level – through the choice of x_i – and product quality:

$$\Pi_i = \underbrace{\frac{q_i u'(q_i \cdot x_i)}{\lambda}}_{p_i} x_i L - vc(x_i L) - F - f(q_i)\,. \tag{4}$$

The first-order condition for maximizing (4) with respect to x_i is[11]

[8] I will use Epsilon to denote the different elasticities considered below: $\varepsilon_{y,x} \equiv \frac{dy}{dx}\frac{x}{y}$.

[9] With $u(c_i) = c_i^\rho$ we are back in the CDE case, with demand elasticity $s = \frac{1}{1-\rho}$.

[10] Zhelobodko et al. (2012) term the inverse of demand elasticity "relative love for variety": $RLV_i \equiv -\frac{c_i u''(c_i)}{u'(c_i)} = \frac{1}{s_i(c_i)} > 0$. For DDE (IDE), as demand elasticity increases (decreases) across product varieties, so does the equilibrium elasticity of substitution and, thereby, the consumer's gain from ("love for") having more varieties increases (decreases).

[11] The profit-maximizing quantity and quality are denoted with an asterisk superscript.

$$q_i \cdot u'\left(q_i \cdot x_i^*\right) + x_i^* \cdot q_i^2 u''\left(q_i \cdot x_i^*\right) = \lambda mc(x_i^* L)\,. \tag{5}$$

Combining condition (5) with the inverse demand function (2) yields the standard monopolistic pricing rule:

$$p_i^* = \frac{mc(x_i^* L)}{1 + \frac{c_i u''\left(c_i^*\right)}{u'\left(c_i^*\right)}} = \mu_i^* mc(x_i^* L)\,. \tag{5a}$$

Where $\mu_i \equiv \frac{s_i(c_i)}{s_i(c_i)-1}$ corresponds to the markup over the marginal cost, and is also the inverse of the revenue elasticity with respect to quantity and quality: $\mu^e = \frac{1}{\varepsilon^e_{R,x}} = \frac{1}{\varepsilon^e_{R,q}}$.

For DDE (IDE), the profit-maximizing markup μ_i increases (decreases) with individual consumption level c_i. The profit-maximizing quality satisfies the following first-order condition:

$$\frac{u'\left(q_i^* \cdot x_i\right) + x_i \cdot q_i^* u''\left(q_i^* \cdot x_i\right)}{\lambda} x_i L = f'(q_i^*)\,. \tag{6}$$

For DDE preferences the marginal revenue decreases with output and with quality - on the right side of (5) and (6), respectively - which is necessary for the existence of a solution to these first order conditions Having $\frac{c_i u'''(c_i)}{u''(c_i)} > -2$ guarantees decreasing marginal revenue functions also for IDE, and that the second-order conditions for profit maximization (over the Hessian matrix) are satisfied.

Combining condition (6) with (5) reveals a positive relation between the profit-maximizing product quality and total and per-consumer supply:

$$Lx_i^* mc(x_i^* L) = q_i^* f'(q_i^*) \Leftrightarrow \frac{y_i^*}{q_i^*} = \frac{f'(q_i^*)}{mc(y_i^*)} \Leftrightarrow \frac{tc(y^*)}{f(q^*)} = \frac{\varepsilon_{f,q^*}}{\varepsilon_{tc,y^*}}\,. \tag{7}$$

Condition (7) implies that profit maximization requires equalizing the relative provision of output and quality to their inverse relative marginal costs, and that output and product quality are positively related regardless of demand characteristics.

Applying (7) we can write the individual per-variety consumption level as:

$$c_i \equiv q_i x_i = \frac{\left(q_i^*\right)^2 f'(q_i^*)}{mc(x_i^* L)L}\,. \tag{7a}$$

Substituting the profit-maximizing price (5a) into (4) yields the following zero-profit expression:

$$\mu_i mc(x_i L) x_i L = vc(x_i L) + f(q_i) + F\,. \tag{8}$$

The profit-maximization condition (7) implies that the free-entry equilibrium also satisfies $\varepsilon^e_{C,y} = \varepsilon^e_{C,q}$, and $\frac{1}{\varepsilon^e_{C,y}} = \frac{1}{\varepsilon^e_{f,q}} + \frac{1}{\varepsilon^e_{tc,y}}$.

Therefore, the symmetric free-entry equilibrium satisfies:[12,13]

$$\mu^e \equiv \frac{s(c^e)}{s(c^e)-1} = \frac{1}{\varepsilon^e_{f,q}} + \frac{1}{\varepsilon^e_{tc,y}}\,. \tag{9}$$

Equation (9) states that in equilibrium the sum of the inverse cost elasticities is equal to the markup (which is also the inverse of the revenue elasticity $\mu^e = \frac{1}{\varepsilon^e_{R,x}} = \frac{1}{\varepsilon^e_{R,q}}$). By (7), both sides of (9) can be implicitly expressed in terms of product quality, with $\frac{dc}{dq}, \frac{dy}{dq} > 0$. Therefore, the solution for (9) is defined by the equilibrium product quality and corresponding equilibrium markups. Let us denote the right side of (9) as $\phi(q) \equiv \frac{1}{\varepsilon^e_{f,q}} + \frac{1}{\varepsilon^e_{tc,y(q)}}$.

For DDE (IDE) preferences the left side of (9) increases (decreases) with q: $\frac{d\mu}{dq} > 0$ ($\frac{d\mu}{dq} < 0$). Under the assumed general innovation cost and production cost functions the sign of $\frac{d\phi(q)}{dq}$ is indefinite. The profit-maximization conditions require that the markup curve crosses the inverse elasticities curve from below. Therefore, for DDE and $\frac{d\phi}{dq} > 0$, the $\mu(q)$ curve must be steeper than $\phi(q)$ around the equilibrium point – $\frac{d\phi}{dq^e} < \frac{d\mu^e}{dq^e}$ – for a solution to exist. Similarly, for IDE and CDE preferences, having $\frac{d\phi(q)}{dq} < 0$ and $\left|\frac{d\phi}{dq^e}\right| > \left|\frac{d\mu^e}{dq^e}\right|$ is necessary for a solution to (9) to exist. For a unique equilibrium to exists the markup function and the sum of inverse elasticity function must intersect exactly once, which requires finer assumption with regard to their functional forms that I assume to be satisfied.

For example, with a constant elasticity of the variable cost – $\varepsilon_{vc,y} = \eta$ – condition (9) yields the explicit form[14]

$$\mu^e \equiv \frac{s(c^e)}{s(c^e)-1} = \frac{1}{\varepsilon^e_{f,q}} + \frac{F}{q^e f'(q^e)} + \frac{1}{\eta}\,. \tag{9a}$$

with $c^e = \frac{q^e}{L}\left(\frac{q^e f'(q^e)}{\eta\cdot\theta}\right)^{\frac{1}{\eta}}$. This concrete expression for c – and equation (7a) for the general case – imply that a market size, L, works to decrease individual consumption level from each variety for a given product quality level. Therefore, with DDE (IDE), an increase in market size shifts the left side of (9) downward (upward). Conse-

[12] The symmetric-equilibrium outcomes are denoted with the "*e*" superscript, and the product variety index "*i*" is dropped.

[13] Having $\varepsilon^e_{f,q} + \varepsilon^e_{vc,y} > \varepsilon^e_{vc,y}\varepsilon^e_{f,q} \iff \frac{\varepsilon^e_{f,q}}{\varepsilon^e_{f,q}-1} > \varepsilon^e_{vc,y}$ is necessary for the right side of (9) to be greater than one and thereby secure positive equilibrium markups.

[14] For $vc(y) = \theta y^\eta$, $\eta \geq 1, \theta > 0$.

quently, the intersection between the $\mu(q)$ and $\phi(q)$ curves shift to the right (left): The equilibrium product quality q^e increases (decreases). The corresponding vertical shift of the intersection (equilibrium) point, which determines the change in equilibrium markup, depends on the slope of $\phi(q)$.

For IDE, the existence of equilibrium requires $\frac{d\phi(q)}{dq} < 0$, and therefore a market size increase necessarily works to increase equilibrium markups: In this case, markups and product quality move in opposite directions due to market size changes. For DDE, the sign of the market size effect on equilibrium markups corresponds to the sign of $\frac{d\phi(q)}{dq}$: Product quality and markups may change in the same or opposite directions due to market size changes.

Proposition 1 *For DDE (IDE) equilibrium product quality increases (decreases) with market size. For DDE (IDE), the market size effect on equilibrium markups is ambiguous and depends on the shape of the elasticities of the production cost and innovation cost (positive).*

As is true of studies of cost reduction innovation, Vives (2008) and Bykadorov and Kokovin (2017) assume constant marginal cost of production, and therefore a unit elasticity of variable production cost. Consequently, their results highlight only the role of the innovation cost elasticity, $\varepsilon^e_{f,q}$, in determining market outcomes. Equation (9) highlights the role of production cost elasticity along with innovation cost elasticity in determining the effect of market size on equilibrium markups for more general production-cost functions.

Furthermore, a departure from the constant-marginal-cost assumption and more generally constant elasticity of the variable cost, $\varepsilon_{vc,y}$, changes the implications of marginal cost reduction to market outcomes: A constant elasticity of the variable cost – η, which is derived from $vc(y) = \theta y^\eta$ – is not affected by reduction in the marginal cost parameter θ. In this case, a reduced marginal cost affects only the individual consumption level – $c = \frac{(q)^2 f'(q)}{mc(xL)\cdot L}$ – just like a decrease in market size: For DDE (IDE), the equilibrium product quality decreases (increases) with the marginal cost and the change in equilibrium markups depends on the shape of ϕ (is negative).

However, the variable cost function $vc(y) = \theta y + y^\eta$, for example, implies the variable cost elasticity $\varepsilon_{vc,y} = 1 + \frac{\eta-1}{\theta y^{1-\eta}+1}$ that increases with a reduction in the marginal cost parameter θ, and therefore would shift also the right side of equation (9), possibly altering the equilibrium changes that were described above.

The latter insights are highlighted in the following proposition:

Proposition 2 *For $\frac{d\varepsilon_{vc,y}}{dmc} = 0$ and DDE (IDE), a lower marginal cost decreases (increases) equilibrium product quality, and the effect on equilibrium markups depends on the sign of $\frac{d\varphi}{dq}$ (is negative). For $\frac{d\varepsilon_{vc,y}}{dmc} \neq 0$, the effect of the marginal cost on equilibrium markups is ambiguous.*

Next, notice that a higher fixed cost F decreases the cost elasticity for any output and product quality levels: This shifts $\phi(q)$ upward, thereby increasing equilibrium product quality. The effect of increased fixed cost on the equilibrium markups depends on demand type: for DDE (IDE) a higher fixed cost implies a lower (higher) equilibrium markups. Thus, for DDE (IDE) product quality and markups change in the same (opposite) direction due to changes in fixed cost.

The latter results are summarized in the following proposition:

Proposition 3 *Equilibrium product quality increases with a higher fixed cost. For DDE (IDE) equilibrium markups increase (decrease) with a higher fixed cost.*

Subject to the equilibrium condition (9), the labor resources-uses constraint defines the product variety span that is offered in the market, which is also the number of operating firms:

$$L = m^e \left[vc(y^e) + F + f(q^e)\right] \Longrightarrow m^e = \frac{L}{vc(y^e) + F + f(q^e)}. \tag{10}$$

As condition (7) already established the firms' optimal output and product quality provision, equation (10) implies the following Corollary - based on Propositions 1–2:

Corollary 1 *For DDE (IDE), market concentration increases (decreases) with market size, as the number of operating firms increases less (more) than proportionally with market size, so each firm serves a larger (smaller) share of the market. A higher fixed cost decreases the number of operating firms.*

With DDE, the effect of market size on the number of operating firms and product varieties that are offered in the market is ambiguous. To see this, combine (10) with (9) to obtain

$$m^e = \frac{L}{\mu^e q^e f'(q^e)}. \tag{11}$$

Recall that the market size effect on equilibrium quality works through shifting the μ^e in equation (9): through changing the individual consumption level, $c_i^e = \frac{(q_i)^2 f'(q_i)}{Lmc(x_i L)}$. Therefore, with DDE, the value of $\frac{(q_i^e)^2 f'(q_i^e)}{L}$ must decrease in the new equilibrium, which implies that the factor $\frac{(q_i^e) f'(q_i^e)}{L}$ in (11) also decreases. This inference implies that if the equilibrium markup does not increase with market size, the number of operating firms increases. However, for the increasing markups, as in the case with DDE, the total effect of market size on the number of operating firms and product variety is ambiguous. This result is presented also in Vives (2008) and Bykadorov and Kokovin (2017).

Tables 1 and 2 summarize all of the results obtained above and are followed by their intuitive economic explanation.

Table 1 Market-Size effects

Demand type	DDE			IDE	CDE
Costs-structure	$\frac{d\phi}{dq} > 0$	$\frac{d\phi}{dq} < 0$	$\frac{d\phi}{dq} = 0$	$\frac{d\phi}{dq} < 0$	$\frac{d\phi}{dq} < 0$
$\frac{\partial q^e}{\partial L}$	> 0	> 0	> 0	< 0	$= 0$
$\frac{\partial \mu^e}{\partial L}$	> 0	< 0	$= 0$	> 0	$= 0$
$\frac{\partial m^e}{\partial L}$	$\lesseqgtr 0$	> 0	> 0	> 0	> 0

Table 2 Fixed cost effects

Demand type	DDE	IDE	CDE
$\frac{\partial q^e}{\partial F}$	> 0	> 0	> 0
$\frac{\partial \mu^e}{\partial F}$	> 0	< 0	$= 0$
$\frac{\partial m^e}{\partial F}$	< 0	< 0	< 0

With a larger market size, demand for each operating firm increases, which by equations (6)–(7) increases the optimal investment in product quality and individual consumption of each variety. For DDE (IDE), the higher individual consumption level per- variety works to increase (decrease) markups. For DDE the total increase in firms' costs is more than proportional to the increase in market size, and even outweighs the market size increase, which pushes some firms out of the market.

With IDE, the increased individual consumption implies decreased markups, and therefore a smaller increase in quality and output, which gives room to more new entering firms. This market entry decreases individual and total demand for each variety. This further erodes the incentive for investment in quality and total supply and thereby works to increase markups. The latter market entry effect dominates the change in equilibrium quality, and the total effect on the markups depends on the shape of the cost elasticity.

A higher fixed cost decreases profitability for the operating firms, which pushes some of them out of the market, and thereby increases demand for the remaining firms. This increase in demand generates a response of an increase in output supply and product quality provision: Under symmetry, the value of the Lagrange multiplier – $\lambda = m \cdot c \cdot u'(c)$ – decreases as firms exit the market, which by equation (6) increases optimal investment in quality and by (7) increases optimal output accordingly. The increase in product quality per-variety output results in a higher per-variety individual consumption: c. For DDE (IDE) the higher individual per-variety consumption implies higher (lower) markups.

In an early version of this study, it is shown that the welfare maximizing market outcomes should satisfy: $\frac{1}{\varepsilon_{u,c}} = \frac{1}{\varepsilon_{tc,y}} + \frac{1}{\varepsilon_{f,q}}$.[15] This implies that if the subutility elasticity – $\varepsilon_{u,c} \equiv \frac{u'(c)}{c \cdot u(c)}$ – decreases with consumption level, the market equilibrium results in the under-provision of product quality and quantity and the over provision of product variety – excessive market entry – as compared with the social optimum. The converse is true if the elasticity of the subutility increases with consumption.

[15] The SSRN working paper, can be found at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4688297.

Last, consider the possibility that a higher-quality product is produced with a higher marginal cost: $vc = vc(y, q)$ and $\frac{\partial^2 vc}{\partial y \partial q} > 0$, as in Bertoletti and Etro (2017) and Fan et al. (2020), though their analyses abstract from innovation cost.[16] In this case, the equilibrium condition (9) generalizes to:

$$\mu^e = \frac{1 - \frac{\varepsilon^e_{vc,q}}{\varepsilon^e_{vc,y}}}{\varepsilon^e_{f,q}} + \frac{1}{\varepsilon^e_{tc,y}} . \tag{12}$$

Equation (12) implies the following proposition:

Proposition 4 *With $\varepsilon^e_{vc,q} > 0$ (marginal cost of production increases with product quality), equilibrium product quality is lower and for DDE (IDE) markups are lower (higher) compared to the benchmark case of $\varepsilon^e_{vc,q} = 0$.*

The results that are presented in Tables 1 and 2 still hold for the generalized function $\phi(q) \equiv \frac{1}{\varepsilon^e_{f,q}} \left(1 - \frac{\varepsilon^e_{vc,q}}{\varepsilon^e_{vc,y}}\right) + \frac{1}{\varepsilon^e_{tc,y}}$.

To illustrate the main results that have been derived thus far, consider the following example of a VDE subutility function:

$$u\left(c_{i,t}\right) = \frac{\left(A + Bc_{i,t}\right)^{\frac{B-1}{B}}}{B - 1} . \tag{13}$$

For $B \neq 1$ and $A = 0$, the subutility function (13) reduces to the familiar *CDE* specification. With $B \neq 1$ and $A \neq 0$, demand elasticity and the implied markup are:[17]

$$\begin{aligned} s(c) &\equiv -\frac{u'(c)}{cu''(c)} = \frac{A}{c} + B \, ; \; \textit{and} \\ \mu &= \frac{\frac{A}{c} + B}{\frac{A}{c} + B - 1} . \end{aligned} \tag{13a}$$

Notice that as consumption approaches infinity, all of the measures that were presented above converge to their *CDE* specification value that is defined by $B > 1$. Consider the case with $A > 0$, for which demand elasticity decreases with consumption level, and ranges from infinity to B. For cost functions, consider the following quadratic forms:

[16] Whereas their setup seems to be an adequate representation of producers' choice from an existing (already developed) set of products, it lacks the notion of the costly innovation process that is associated with developing improved product quality.

[17] For $B = -1$ the utility function (13) takes the quadratic form $u\left(c_{i,t}\right) = -\left(A - c_{i,t}\right)^2$, and for $B = 1$ it converges to the logarithm form $u\left(c_{i,t}\right) = \ln\left(A + c_{i,t}\right)$.

$$
\begin{aligned}
tc(q) &= F + \theta y^2 \Rightarrow \varepsilon_{tc,y} = \frac{2}{\frac{F}{\theta y^2} + 1}\,; \; and \\
f(q) &= \phi q^2 \Rightarrow \varepsilon_{f,q} = 2\,.
\end{aligned}
\tag{14}
$$

By equation (7), the assumed cost functions imply $xL = q\sqrt{\frac{\phi}{\theta}}, c = \frac{q^2}{L}\sqrt{\frac{\phi}{\theta}}$, and the optimality condition (9) is:

$$
\frac{\frac{A}{\frac{q^2}{L}\sqrt{\frac{\phi}{\theta}}} + B}{\frac{A}{\frac{q^2}{L}\sqrt{\frac{\phi}{\theta}}} + B - 1} = \frac{F}{2q^2\phi} + 1\,. \tag{15}
$$

The left side in (15) increases from one to $\frac{B}{B-1}$, and the right side decreases from infinity to 1; therefore a unique equilibrium quality is guaranteed to exist, with a corresponding equilibrium markup $\mu > 1$. An increase in market size – L – shifts the left side of (15) downward, which thereby reduces equilibrium markups and increases product quality; a decrease in the marginal cost parameter, θ, works in the opposite direction.

Applying the assumed cost functions to (11) yields $m^e = \frac{L}{F+2\phi q^2} \Rightarrow q^2 = \frac{L}{2\phi m^e} - \frac{F}{2\phi}$; and substituting this expression in (15) yields:

$$
\frac{L}{Fm^e} = \frac{A}{\left(\frac{1}{m^e} - \frac{F}{L}\right)\sqrt{\frac{1}{4\theta\phi}}} + B\,. \tag{16}
$$

Equation (16) defines the unique equilibrium number of operating firms (and product variety): m^e. For the assumed subutility and cost functions, product variety increases with market size and marginal cost and decreases with the fixed cost.

4 Cournot Competition

The above analysis of monopolistic competition assumed a sufficiently large number of firms, so that each one of them takes market outcomes as given, which is the common practice in the growth and international trade literature. The market conditions that were taken as given were incorporated in the value of the Lagrange multiplier that was presented in equations (2) and (4)–(6) – $\lambda = \sum_{i=1}^{m} q_i \cdot x_i \cdot u'(q_i \cdot x_i)$ – which works as an aggregate price index that incorporates other firms' decisions over their output and product quality, and affects the demand that is faced by each firm.[18]

When each firm accounts for its own effect on the value of λ, this effect is not negligible anymore. Consequently, conditions (5)–(6) change to

[18] Considering a finite 'small' number of firms here, we switch from integrating to summation over *m*.

$$p=\underbrace{\frac{m}{m-1}\frac{s(c^*)}{s(c^*)-1}}_{\text{modified markup}}mc(x^*L);\ and \tag{17}$$

$$\frac{m-1}{m}\frac{\left[u'\left(q^*\cdot x\right)+q^*xu''\left(q^*\cdot x\right)\right]}{\lambda}xL=f'(q^*)\,. \tag{18}$$

As the number of firms *m* approaches infinity, the profit maximizing conditions (17)–(18) coincide with conditions (5)–(6) that were derived above. The symmetric profit-maximizing price and markup in (17) is higher than under monopolistic competition, and these values decrease with the number of operating firms for a given individual demand elasticity. Condition (18) can be written also as $\frac{m-1}{m^2}\frac{s(c^*)-1}{s(c^*)}xL=f'(q^*)$, which implies that the symmetric profit-maximizing quality decreases with the number of competing firms, for a given individual demand elasticity.

Nonetheless, combining (17) and (18) reveals that the profit-maximizing ratio of output to product quality remains as in condition (7) above, and therefore the free-entry (zero profit) equilibrium condition (9) changes only due to the modified markup expression:

$$\frac{m}{m-1}\cdot\frac{s(c^e)}{s(c^e)-1}=\frac{1}{\varepsilon^e_{f,q}}+\frac{1}{\varepsilon^e_{tc,y}}\,. \tag{19}$$

Combining (19) with the labor resource-uses constraint (10) yields the following equilibrium equation:

$$\frac{1}{1-\frac{vc(y^e)+F+f(q^e)}{L}}\cdot\frac{s(c^e)}{s(c^e)-1}=\frac{1}{\varepsilon^e_{f,q}}+\frac{1}{\varepsilon^e_{tc,y}}\,. \tag{19a}$$

Based on equation (7) the equilibrium condition (19a) can be written in terms of product quality only.[19] The equilibrium condition (19a) defines the same relationship between markups and cost elasticities as does the equilibrium condition (9) for the monopolistic competition. The cost elasticities term on the right side in equation (9) is identical to its counterpart in (19a); however, the left side in (9) depends only on individual demand elasticity while the left side in (19a) depends also – negatively – on the number of competing firms, which is smaller under Courtnot competition (compared with monopolistic competition).

By (7) and (10), the number of competing firms in equilibrium decreases as their individual provision of product quality increases. Therefore, the left side of equation (19a) is smaller than its counterpart in (9), which implies the following proposition:

[19] With constant marginal cost, for example, (19a) reads $\frac{1}{1-\frac{q_i^*f'(q_i^*)+f(q^e)+F}{L}}\frac{s(c^e)}{s(c^e)-1}=\frac{1}{\varepsilon^e_{f,q}}+\frac{F}{q_i^*f'(q_i^*)}+1$, and $c^e=\frac{(q^e)^2f'(q^e)}{mc\cdot L}$.

Proposition 5 *Equilibrium product quality under Cournot competition is smaller than under monopolistic competition. Markups under Cournot competition can be higher (for* $\frac{d\phi}{dq} < 0$*) or lower (for* $\frac{d\phi}{dq} < 0$*) than under monopolistic competition.*

I refer to the effect of the first factor on the left side of (19a) on market outcomes as the "competition factor", which by equations (17)–(18) works to decrease profit-maximizing markups and quality provision as the number of competing firms, m, increases. Due to the labor resources-uses constraint, the value of the competition factor increases with the provision of product quality, and decreases with larger market size and higher marginal cost.

Therefore, for DDE (IDE), the effect of product quality, market size, and marginal cost on the competition factor is the same (contrary) to their effect on the individual demand elasticity which is presented in the second factor on the left side of (19a). Consequently, the results that were presented in Proposition 1 with regard to the market size effect on market outcomes for DDE still hold, and are amplified under Cournot competition; but the market size and marginal cost effects for IDE become ambiguous. As the competition effect diminishes with the number of operating firms, it is more likely to denominate the individual demand elasticity effect when the market is more concentrated.

Bertoletti and Epifani (2014) study the same framework without product quality innovation and with constant marginal cost.[20] Focusing on the effect of market size on equilibrium markups they conclude that for DDE a larger market size (which they associate with globalization) implies lower markups (which they interpret as a pro-competitive effect). The current analysis adds to their results that the change in markups depends on the sign of ϕ, but the effect on equilibrium product quality is positive regardless.

A higher fixed cost works to increase the competition effect factor in the left side of (19a), while also increasing the sum of inverse cost elasticities on the left side (as in the monopolistic competition case). However, substituting the labor-market-clearing condition (8) into the right side of (19a) reveals that for $m > 2$ the initial Fixed cost effect that was defined in Proposition 3 dominates the Coutnot competition effect.

For an illustration the results that were derived in this section, consider the cost functions from (14) and the logarithmic DDE case (with $B = 1$, $A > 0$) with of the subutility (13), for which the equilibrium equation (19a) is:

$$\frac{1}{1 - \frac{F+2\phi q^2}{L}} \left(\frac{\frac{q^2}{L}\sqrt{\frac{\phi}{\theta}}}{A} + 1 \right) = \frac{F}{2\phi q^2} + 1 \,. \tag{20}$$

The left (right) side in (20) increases (decreases) with product quality. A larger market size – L – lowers the left side in (20), which thereby increases equilibrium quality and reduces the equilibrium markup. A higher fixed cost – F – shifts both sides of (20) upward, which implies an ambiguous change in equilibrium quality, but for

[20] See (Bertoletti and Epifani 2014, Subsection 4.2, p. 233).

$m > 2$ the effect of change on the right side is dominant. A decrease in the marginal cost parameter, θ, works like a market size increase.

5 Conclusion

This study adds the analysis of product-quality innovation to the growing literature on monopolistic competition with variable demand elasticity. It characterizes the free-entry equilibrium, and highlights the roles of firms' cost structure and demand type in determining the effects of market size, fixed cost and marginal cost on equilibrium markups and product quality. Furthermore, it is shown how market outcomes and their comparative statics change when there are a small number of big firms that engage in Cournot interaction. Thereby, the theoretical results that are derived here provide a contribution to the ongoing deliberation and research on the apparent increases in market concentration and relaxed competition across American industries. In Sorek (2025) I elaborate the present analysis into a dynamic general equilibrium framework to explore its implications for Schumpeterian growth theory. The possible implications of the market analysis that is presented here to the more-elaborate frameworks of multi-markets and heterogenous firms that have been studied in the international trade and macroeconomics realms, are yet to be explored.

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Declarations

Competing Interests The authors declare no competing interests.

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