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Some metric fixed point theorems for weakly orbitally continuous mappings

Kishan S. Dani 0009-0004-7423-6144,
N. Chandra* 0000-0003-0501-3401,
Mahesh C. Joshi 0000-0001-9222-5925,
S. Rawat 0009-0006-3183-2573

Abstract. The purpose of this paper is to establish some fixed point results on the complete and orbitally complete metric spaces for Ćirić type contractive mappings in the context of weakly orbital continuity. Also, the concept of asymptotically regular mapping is used for the existence of fixed points. However, our results generalize several well known results in the literature. Additionally, we have presented our theorems through some non-trivial examples, and an application to integral equations has also provided.

1. Introduction

The Banach contraction principle (BCP) has been proven by S. Banach [1] in 1922 in metric spaces and further generalized by many researchers. However, various versions of the new contractions obtained in the literature have been obtained according to the metric spaces studied, and provided some useful aspects in the view of applications to many fields, e.g., game theory, mathematical economics, optimization problems, approximation theory, initial and boundary value problem in ordinary and partial differential equations, variational inequality, engineering, and many others. Indeed, since last few decades, some important fixed point results for contractive and non-expansive mappings have been obtained by several authors with more general conditions and their applications in various fields, see [2,3,5–7,9–21] and references therein.

In 1971, Lj. B. Ćirić has been introduced the following notion of orbital continuity as a generalization of continuity.

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*Corresponding Author.

Definition 1 ([10–12])**.** A self-mapping f of a metric space (X,d) is said to be orbitally continuous at $x \in X$ if $\lim_i f^{m_i}x = u$ implies $\lim_i ff^{m_i}x = fu$, and the set $O(x,f) = \{f^n x : n = 0,1,2,\ldots\}$ is called the orbit of f at x. However, X is said to be f-orbitally complete iff every Cauchy of the form $\{f^{n_i}x : i = 1,2,3,\ldots\}$ converges in X.

Note that continuity implies orbital continuity but not conversely, see [9,10]. However, in 2017, Pant et al. [20], introduced another weaker form of continuity as follows.

Definition 2 ([20])**.** A self-mapping in a metric space (X,d) is said to be k-continuous, $k = 1,2,3,\ldots$, if $\lim_n f^k x_n = fu$ whenever $\{x_n\}$ is a sequence in X such that $\lim_n f^{k-1}x_n = u$.

It is interesting to observe that continuity of f^k and k-continuity of f are independent condition when $k > 1$, and *continuity* $\Longrightarrow$ $2-$*continuity* $\Longrightarrow$ $3-$ *continuity* $\Longrightarrow$ $\ldots$, but not conversely, see [20] in details. It is also easy to see that 1-continuity is equivalent to continuity.

Furthermore, in 2019, Pant et al. [21] also introduced a generalization of orbital continuity which is defined as the following.

Definition 3 ([21])**.** A self-mapping f of a metric space (X,d) is said to be weakly orbitally continuous if the set $\{y \in X : \lim_i f^{m_i}y = u \implies \lim_i ff^{m_i}y = fu\}$ is non-empty whenever the set $\{x \in X : \lim_i f^{m_i}x = u\}$ is non-empty.

It is observed that orbital continuity implies weakly orbital continuity but not conversely, and also weakly orbitally continuous mappings may not be k-continuous. For instance, Pant et al. [21] has shown that if $f : X \to X$ is defined by

$$f(x) = \begin{cases} \frac{1+x}{2}, & \text{if } x < 1; \\ 0, & \text{if } 1 \le x < 2; \\ 2, & \text{if } x = 2, \end{cases}$$

where $X = [0,2]$ is equipped with the Euclidean metric. Then $f^n(0) \to 1$ and $f(f^n 0) \to 1 \neq f(1)$, and so f is not orbitally continuous. However, f is weakly orbitally continuous because if we take $x = 2$, $f^n(2) \to 2$ and $f(f^n 2) \to 2 = f(2)$. Also, for the sequence $\{f^n 0\}$, we have $f^{k-1}(f^n 0) \to 1$ and $f^k(f^n 0) \to 1 \neq f(1)$ for any integer $k \geq 1$, i.e., f is not k-continuous.

Definition 4 ([4,18])**.** A self-mapping f of metric space (X,d) is said to be asymptotically regular if, for every $x \in X$, we have $\lim_n d(f^n x, f^{n+1}x) = 0$.

However, Górnicki [18] and Bisht [2] established the following fixed point result for asymptotically regular mappings.

Theorem 1 ([2, 18])**.** *Let f be an asymptotically regular self-mapping of a complete metric space (X, d). Suppose there exist $0 \leq M < 1$ and $0 \leq K < +\infty$ such that*

$$d(fx, fy) \leq M\, d(x, y) + K\{d(x, fx) + d(y, fy)\} \tag{1}$$

for all $x, y \in X$. If f is either k-continuous for some $k \geq 1$ or orbitally continuous, then f has a unique fixed point $z \in X$, and $\lim_n f^n x = z$ for each $x \in X$.

Indeed, the above Theorem 1 does not hold if the condition (1) is not satisfied, see Examples 2.3 and 2.5 in [19] for more details.

Let Δ denote the class of those functions $\alpha : [0, \infty) \to [0, 1)$, which satisfy the condition that $\lim_n \alpha(t_n) = 1 \implies \lim t_n = 0$. Further, let Σ denote the class of those functions $\beta : [0, \infty) \to [0, \infty)$, which satisfy the conditions:

(a) $\beta(t) < t$ for all $t > 0$;

(b) β is upper semi-continuous, i.e.,

$$t_n \to t \geq 0 \implies \limsup_{n\to\infty} \beta(t_n) \leq \beta(t).$$

Recently, Górnicki [19] also established the following fixed point results by replacing the constant M of Theorem 1 by functions of classes Δ and Σ.

Theorem 2 ([19])**.** *Let f be an asymptotically regular self-mapping of a complete metric space (X, d). Suppose there exist $\alpha \in \Delta$ and $0 \leq K < +\infty$ such that*

$$d(fx, fy) \leq \alpha(d(x, y))\, d(x, y) + K\{d(x, fx) + d(y, fy)\} \tag{2}$$

for all $x, y \in X$. If f is either k-continuous for some $k \geq 1$ or orbitally continuous, then f has a unique fixed point $z \in X$, and $\lim_n f^n x = z$ for each $x \in X$.

Theorem 3 ([19])**.** *Let f be an asymptotically regular self-mapping of a complete metric space (X, d). Suppose there exist $\beta \in \Sigma$ and $0 \leq K < +\infty$ such that*

$$d(fx, fy) \leq \beta(d(x, y))\, d(x, y) + K\{d(x, fx) + d(y, fy)\} \tag{3}$$

for all $x, y \in X$. If f is either k-continuous for some $k \geq 1$ or orbitally continuous, then f has a unique fixed point $z \in X$, and $\lim_n f^n x = z$ for each $x \in X$.

Remark 1. Theorem 2 is a particular case of Theorem 3. These results are very important because an asymptotically regular mapping f of a metric space (X, d) may not have a fixed point which satisfies the condition: $d(fx, fy) < d(x, fx) + d(y, fy)$ for all $x, y \in X$ with $x \neq y$, see Example 3.2 in [17].

Besides, in 1976, Ćirić [14] proved the following fixed point results by using the concept of orbital continuity in a metric space.

Theorem 4 ([14]). *Let f be a self-mapping of a metric space (X,d). If X is f-orbitally complete and f is an orbitally continuous mapping satisfying the condition:*

$$d(fx,fy) \leq q \max \left\{ \begin{array}{l} d(x,y), \frac{d(x,fx)d(y,fy)}{d(x,y)}, \\ a(x,y)d(x,fy)d(y,fx) \end{array} \right\} \tag{4}$$

for all $x,y \in X$, $x \neq y$ with $q < 1$, where $a(x,y)$ is a non-negative real function, then f has a fixed point $z \in X$, and $\lim\limits_n f^n x = z$ for each $x \in X$. In addition, if $a(x,y) \leq \frac{1}{d(x,y)}$ then f has a unique fixed point.

Theorem 5 ([14]). *Let f be an orbitally continuous self-mapping of a metric space (X,d). If f satisfies the condition (4) with $q = 1$, and the sequence $\{f^n x_0\}_{n=1}^{\infty}$ has a limit point $z \in X$ for some $x_0 \in X$, then z is a fixed point of f and $\lim\limits_n f^n x_0 = z$.*

It is observed that mappings satisfying (4) with $q < 1$ does not imply the existence of a fixed point of f, even if X is compact and $a(x,y) = 0$, for instance, see the example in [14]. Also, such mappings may have infinitely many or be without fixed point, even though X is compact.

In the next section, we establish some fixed point results on the complete and orbitally complete metric spaces in the context of weakly orbital continuity and asymptotic regularity.

2. MAIN RESULTS

Theorem 6. *Let f be a self-mapping of a complete metric space (X,d) satisfying (4) with $q < 1$. If f is weakly orbitally continuous then it has a unique fixed point $z \in X$. Moreover, $\lim\limits_n f^n x = z$ for each $x \in X$.*

Proof. Let $x_0 \in X$ be arbitrary and we consider a sequence $\{x_n\}$ in X such that $x_n = f^n(x_0) = fx_{n-1}$, where $n \in \mathbb{N}$ (set of positive integers). In case of $x_n = x_{n+1}$ for some $n \in \mathbb{N}$, we get $x_n = x_{n+1} = x_{n+2} \ldots$, i.e., $\{x_n\}$ is a Cauchy sequence, and obviously x_n is a fixed point of f. So, we assume $x_n \neq x_{n+1}$ for any n. Then, we have

$$\begin{aligned} d(x_1,x_2) &= d(fx_0,fx_1) \\ &\leq q \max \left\{ \begin{array}{l} d(x_0,x_1), \frac{d(x_0,fx_0)d(x_1,fx_1)}{d(x_0,x_1)}, \\ a(x,y)d(x_0,fx_1)d(x_1,fx_0) \end{array} \right\} \\ &= q \max\{d(x_0,x_1), d(x_1,x_2)\}. \end{aligned}$$

Hence, $d(x_1, x_2) \leq q\, d(x_0, x_1)$, because $q < 1$. Also, we get

$$\begin{aligned} d(x_2, x_3) &= d(fx_1, fx_2) \\ &\leq q \max\left\{ d(x_1, x_2), \frac{d(x_1, fx_1) d(x_1, fx_2)}{d(x_1, x_2)} \right\} \\ &= q \max\{d(x_1, x_2) d(x_2, x_3)\}. \end{aligned}$$

That is, $d(x_1, x_2) \leq qd(x_1, x_2)$. In a similar manner, we can find that $d(x_n, x_{n+1}) \leq q^n d(x_0, x_1)$ for all n. Since $q < 1$, we get $\lim\limits_{n\to\infty} d(x_n, x_{n+1}) = 0$. Now, for any $p \in \mathbb{N}$, we have

$$d(f^n x_0, f^{n+p} x_0) \leq \sum_{k=0}^{p-1} d(f^{n+k} x_0, f^{n+p+1} x_0).$$

Thus, we obtain the following inequality

$$d(f^n x_0, f^{n+p} x_0) \leq \sum_{k=0}^{p-1} q^{n+k} d(x_0, fx_0),$$

that is,

$$d(f^n x_0, f^{n+p} x_0) \leq q^n d(x_0, fx_0) \sum_{k=0}^{p-1} q^k.$$

Since $\sum_{k=0}^{p-1} q^k$ is a geometric series, then holds $\sum_{k=0}^{p-1} q^k = \frac{1-q^p}{1-q} < \frac{1}{1-q}$. Thus, $d(f^n x_0, f^{n+p} x_0) \leq \frac{q^n}{1-q} d(x_0, fx_0)$, and so $\lim\limits_{n} d(f^n x_0, f^{n+p} x_0) = 0$. Hence, $\{x_n\}$ is a Cauchy sequence, and the completeness of X implies that there exists $z \in X$ such that $\lim\limits_{n} x_n = z$. Indeed, for each $p \geq 1$, we have $\lim\limits_{n} f^{n+p} x_0 = z$, particularly $\lim\limits_{n} f^n x_0 = z$ for any y in X. Furthermore, if f is weakly orbitally continuous then we have, $\lim\limits_{n} f^n y_0 = z$ and $\lim\limits_{n} f^{n+1} y_0 = fz$ for some $y_0 \in X$. Therefore $z = fz$, i.e., z is a fixed point f. However, the uniqueness of fixed point follows easily. $\square$

Next, we see the following example to illustrate the obtained result.

Example 1. Let a self mapping f of a Euclidean metric space $X = [0, 1]$ be defined by

$$T(x) = \begin{cases} \frac{x}{4}, & \text{if } x \in [0, 1); \\ 0.8, & \text{if } x = 1. \end{cases}$$

Then, f is not orbitally continuous. If we consider a sequence $\{x_n\} \subset [0, 1)$ with $x_n = 1 - \frac{1}{n}$, then $\{x_n\} \to 1$, but $fx_n \to \frac{1}{4} \neq f1$. Nevertheless, f is weak orbitally continuous because f is continuous on $[0, 1)$. Moreover, f satisfies the condition (4) with $q < 1$ and $0 \in X$ is the only fixed point of f.

Theorem 7. *Let f be a self-mapping of a complete metric space (X,d) satisfying (4) with $q=1$ and the sequence $\{f^n x_0\}_{n=1}^{\infty}$ has a limit point $z \in X$ for some $x_0 \in X$, then z is a fixed point of f and $\lim_n f^n x_0 = z$ provided that f is weakly orbitally continuous.*

Proof. Following the process mentioned previously in Theorem 6, if $fx_{n-1} = fx_n$ for some $n \in \mathbb{N}$ then the assertion obviously follows. Assume now that $fx_{n-1} \neq fx_n$ for all n and let $\lim_n f^n x_0 = z$. Then, by (4) with $q=1$, we have

$$\begin{aligned} d(ffx_{n-1}, ffx_n) &< \max\left\{ d(fx_{n-1}, fx_n), \frac{d(fx_{n-1}, ffx_{n-1})d(fx_n, ffx_n)}{d(fx_{n-1}, fx_n)} \right\} \\ &= \max\{d(fx_{n-1}, fx_n), d(fx_n, fx_{n+1})\}, \end{aligned}$$

hence $d(fx_n, fx_{n+1}) < d(fx_{n-1}, fx_n)$. Thus $\{d(fx_n, fx_{n+1})\}_{n=0}^{\infty}$ is a decreasing sequence of positive reals, so it is convergent.

As $\lim_i f^{n_i} x_0 = z$ and f is weakly orbitally continuous, it follows that that $\lim_i f^{n_i} x_0 = z$, $\lim_i f^{n_i+1} x_0 = fz$, $\lim_i f^{n_i+2} x_0 = f^2 z$, and so

$$\lim_i d(f^{n_i} x_0, f^{n_i+1} x_0) = d(z, fz)$$

and

$$\lim_i d(f^{n_i+1} x_0, f^{n_i+2} x_0) = d(fz, f^2 z).$$

Since

$$\{d(f^{n_i} x_0, f^{n_i+1} x_0)\}_{i=1}^{\infty} \quad \text{and} \quad \{d(f^{n_i+1} x_0, f^{n_i+2} x_0)\}_{i=1}^{\infty}$$

are sub-sequences of the convergent sequence $\{d(f^n x_0, f^{n+1} x_0)\}_{n=0}^{\infty}$, we have

$$d(z, fz) = d(fz, f^2 z).$$

Now, if we assume that $z \neq fz$ then by (4) (with $q=1$), we find

$$d(fz, f^2 z) < d(z, fz),$$

a contradiction. Hence $fz = z$, i.e., z is a fixed point of f. □

Additionally, we have the following corollary of our results.

Corollary 1. *Let (X,d) be a metric space and $f : X \to X$ be a one one, continuous and subsequentially convergent mapping. Let g be a self mapping of X which satisfying the following:*

$$d(fg(x), gf(y)) \leq q \max \left\{ \begin{array}{l} d(fx, fy), d(fx, fy)d(fx, fgx)(fy, fgy), \\ \alpha(x,y)d(fx, fgy)d(fy, fgx) \end{array} \right\}$$

for every $x, y \in X$ with $x \neq y$ and $0 < q < 1$, where $\alpha(x,y)$ is a real non-negative function. If every sequence is a Cauchy sequence of the form $\{fg^{n_i} x\}$ is convergent and g is a weakly orbitally continuous mapping, then $\lim_{n\to\infty} f^n x = z \in X$ for each $x \in X$, and $gz = z$.

Theorem 8. *Let f be an asymptotically regular self-mapping of a complete metric space (X, d). Suppose there exist $\beta \in \Sigma$ and $0 \leq K < +\infty$ such that f satisfies the condition (3) for all $x, y \in X$. Further, if f is weakly orbitally continuous then f has a unique fixed point.*

Proof. Let $x_0 \in X$ and let consider a sequence $\{x_n\}$ defined by $x_n = f^n(x_0)$, where $n \in \mathbb{N}$. Now, if $f^p(x_0) = f(f^p(x_0))$ then $f^p(x_0)$ is the fixed point of f. So, we assume that $f^n(x_0) \neq f^{n+1}(x_0)$ for all $n \in \mathbb{N} \cup \{0\}$. Then, we can find $m_i \geq n_i > i$ and $d(x_{n_i}, x_{m_i}) \geq \epsilon$ for $i \in \mathbb{N}$. Also, we can choose m_i so small as possible such that $d(x_{n_i}, x_{m_{i-1}}) < \epsilon$. Hence, for each $i \in \mathbb{N}$, we have

$$\epsilon \leq d(x_{n_i}, x_{m_i}) \leq d(x_{n_i}, x_{m_{i-1}}) + d(x_{m_{i-1}}, x_{m_i}) < \epsilon + d(x_{m_{i-1}}, x_{m_i}).$$

And it follows from asymptotic regularity that $\lim\limits_{i \to \infty} d(x_{n_i}, x_{m_i}) = \epsilon$. However,

$$\begin{aligned} d(x_{n_i}, x_{m_i}) &\leq d(x_{n_i}, x_{n_{i+1}}) + d(x_{n_{i+1}}, x_{m_{i+1}}) + d(x_{m_{i+}}, x_{m_i}) \\ &\leq d(x_{n_i}, x_{n_{i+1}}) + d(x_{m_i}, x_{m_{i+1}}) + \beta(d(x_{n_i}, x_{m_i})) \\ &\quad +K\{d(x_{n_i}, x_{n_{i+1}}) + d(x_{m_i}, x_{m_{i+1}})\} \\ &= \beta\, d(x_{n_i}, x_{m_i}) + (K+1)\{d(x_{n_i}, x_{n_{i+1}}) + d(x_{m_i}, x_{m_{i+1}})\}. \end{aligned}$$

Thus, making $i \to \infty$ and due to asymptotic regularity of f, we have

$$d(x_{n_i}, x_{n_{i+1}}) = 0 = d(x_{m_i}, x_{m_{i+1}}).$$

Since β is upper semi continuous, we get

$$0 < \epsilon = \lim_{i \to \infty} d(x_{n_i}, x_{m_i}) \leq \lim_{i \to \infty} \sup \beta(d(x_{n_i}, x_{m_i}) \leq \beta(\epsilon) < \epsilon$$

which is a contradiction. Hence $\{x_n\}$ is a Cauchy sequence, and completeness of the space X implies that it converges to a limit, say z. That is, $\lim\limits_{n \to \infty} x_n = z \in X$. Further, f is weakly orbitally continuous and $f^n(x) \to z$, therefore $f^{n_i+1}(x) \to f(z)$ for some subsequence n_i. On the other hand, we have $f^{n_i+1}(x) \to z$ because the sequence $f^n(x)$ converges to z. Hence, $f(z) = z$, i.e., z is a fixed point of f. However, the uniqueness of fixed point is obvious. □

Furthermost, we provide an example to demonstrate the Theorem 8 as follows.

Example 2. Let $X = [0, 2]$ be a usual metric space. We define a function f on X as:

$$f(x) = \begin{cases} 0, & 0 \leq x < 1, \\ \frac{1}{2}, & 1 \leq x \leq 2. \end{cases}$$

Then, by choosing $\beta(t) = 0$ for all $t \geq 0$ and $K = 1$, we have (3) is satisfied for all $x, y \in X$. Also, if $x \in [0, 1)$ then $f(x) = 0$ and $f^n(x) = 0$ for all

$n \geq 1$, and if $x \in [1,2]$, then $f(x) = \frac{1}{2}$ and $f^2(x) = f(\frac{1}{2}) = 0$ which gives $f^n(x) = f^{n+1}(x) = 0$.
Hence $\lim_{n\to\infty} d(f^n(x), f^{n+1}(x)) = 0$, i.e., f is asymptotic regular.
However, f is weakly orbitally continuous because if $x \in [0,1)$, then $f^n(x) = 0$ for all $n \geq 1$, and so $\lim_{k\to\infty} f(f^{n_k}) = 0 = f(x)$. Obviously, 0 is the only fixed point of f.

Remark 2. It is easy to see that our results are the generalizations of Theorem 1, Theorem 2, Theorem 3, Theorem 4, and Theorem 5, which are mentioned in the previous section.

3. AN APPLICATION

The use of fixed point theorem has been developed in many fields, particularly to determine the specific solution of differential and integral equations. Previously, fixed point theorems have been used to show the existence of solution of the Volterra integral equation in the demographic field of viscoelastic material, and in the mathematical insurance of renewed equations. An approximate solution to the Volterra integral equation has been also published in the literature, see [8,22,24,25] and references therein.

Now, we consider a Volterra integral equation given by Shibombing et al. [23] and find a solution by using the obtained results. Note that the general form of a Volterra integral equation is as follows:

$$x(t) = f(t) + \mu \int_a^t k(t,s)x(s)ds \tag{5}$$

Thus, by choosing $f(t) \in [a,b]$, $\mu \in [0,1)$ as a initial function $x_0(t)$, one can introduce a fixed point iteration with the integral equation

$$f(x) = g(t) + \mu \int_a^t k(t,s)x(s)ds,$$

where $k(t,s)$ is a continuous function on $\mathcal{R}$ such that

$$\mathcal{R} = \{(t,s) | a < s < t, a < t < b\}.$$

Here we define an operator f such that

$$f(x) = g(t) + \mu \int_a^t k(t,s)x(s)ds. \tag{6}$$

We discuss the integral equation located in the space $C[a,b]$, i.e., the space of all continuous functions define on interval $I = [a,b]$ with the metric given by

$$d(x,y) = \lim_{t\in I} |x(t)| - y(t)|. \tag{7}$$

Theorem 9. *Let $f(x)$ in equation* (6) *be a continuous function in* $[a,b]$ *and* k *is the continuous kernel on the region* $\mathcal{R}$ *in the plane* $\mathbb{R}^2$ *with* $a \leq s \leq t, a \leq t \leq b$ *such that* $k(x,t) < c$. *Then, the equation* (6) *has a solution.*

Proof. Let $f : C[a,b] \to C[a,b]$ be a continuous function in the integral equation (6), where $k(t,s)$ is continuous and $k(t,s) \leq c$. Now, by using contractive condition (4), we find the solution of the integral equation (6). First, we note that

$$\begin{aligned} d(fx,fy) &= \lim_{t\in I}\left|\left(g(t)+\mu\int_a^t k(t,s)x(s)\right)-\left(g(t)+\mu\int_a^t k(t,s)y(s)ds\right)\right| \\ &= \lim_{t\in I}\left|\mu\int_a^t k(t,s)x(s)ds-\mu\int_a^t k(t,s)y(s)ds\right| \\ &= |\mu|\lim_{t\in I}\left|\int_a^t k(t,s)[x(s)-y(s)]ds\right| \\ &\leq |\mu|\, c\, d(x,y)\,(t-a). \end{aligned}$$

As the right side of above inequality always contains the term $d(x,y)$, i.e., in $d(fx,fy) \leq qd(x,y)$, where $q = |\mu|c\,(t-a)$ in such way that $0 \leq q < 1$. Hence the Volterra integral equation can be solved by the condition (4). □

Compliance with ethical standards

Conflict of interest. The authors declare that they have no conflict of interest.

Ethical approval. This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent. Informed consent was obtained from all individual participants included in the study.

Use of AI tools declaration. The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Kishan S. Dani
Department of Mathematics
Kumaun University, DSB Campus
Nainital-263002, Uttarakhand
India
E-mail address: `kd149dani@gmail.com`

N. Chandra
Department of Mathematics
HNB Garhwal University, BGR Campus
Pauri-246001, Uttarakhand
India
E-mail address: `cnaveen329@gmail.com`

Mahesh C. Joshi
Department of Mathematics
Kumaun University, DSB Campus
Nainital-263002, Uttarakhand
India
E-mail address: `mcjoshi69@gmail.com`

S. Rawat
Department of Mathematics
HNB Garhwal University, BGR Campus
Pauri-246001, Uttarakhand
India
E-mail address: `srw20596@gmail.com`