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Incentive mechanism of rural environmental governance based on a public–private partnership (PPP) model

Cui-ling Gu[1*], Yi-xian Li[1], Xiao-jian Ma[1], Jinhua Zhao[2] and Rui Ding[3]

Abstract

Rural environmental governance is the core link of rural revitalization. However, in practice, this model faces problems, such as a shortage of funds, unclear rights and responsibilities, and insufficient motivation. Therefore, by introducing a public–private partnership (PPP) model and based on principal-agent theory, this paper explores how to effectively design an incentive mechanism that will promote rural environmental governance. Firstly, a principal-agent incentive contract model is established, based on the PPP model, in which local governments are the principals and private enterprises are the agents. The analysis results show that, by reducing the risk aversion degree of private enterprises and improving their marginal output benefit coefficient, the investment intensity and effort level in environmental governance can be significantly enhanced. Secondly, in order to incorporate the participation of rural residents, the original model is expanded, and a PPP model principal-agent incentive contract model covering local governments, private enterprises, and rural residents is constructed. The analysis of this model shows that, compared with the non-cooperative model, the environmental governance model with the collaborative participation of local governments and rural residents is superior. Under the cooperative framework jointly constructed by local governments and rural residents, the enthusiasm of private enterprises to participate in environmental governance can be significantly improved by reducing the uncertainty of the external environment, alleviating the risk aversion degree of private enterprises, reducing the cost of environmental governance, and enhancing the overall efficiency of environmental governance. The research conclusions can provide scientific decision-making references for local governments when promoting rural environmental governance by applying the PPP model.

Keywords Rural environmental governance, PPP model, Principal-agent theory, Incentive contract

1 Introduction

A global consensus on sustainable development has been developing, while China's ecological civilization construction and rural revitalization strategy continue to advance (Shen et al 2024). Promoting rural construction constitutes a key task in implementing China's rural revitalization strategy (Song and Su 2025). The "No. 1 Central Document" of 2024 explicitly emphasizes strengthening the rural ecological civilization, advancing the campaign against agricultural and rural pollution, and integrating rural ecological protection and restoration. As the foundation of rural ecology and residents' livelihoods, the rural human settlement environment directly influences green-low-carbon rural development and residents' physical and mental health (Wang et al 2023). As urbanization continues to progress, rural economies continue to grow significantly. Unfortunately, industrial expansion

*Correspondence:
Gu Cui-ling
gucui2020@163.com
[1] School of Management, Henan University of Technology, Henan 450001, China
[2] Faculty of Artificial Intelligence in Education, Central China Normal University, Wuhan 430079, China
[3] School of Economics and Management, Wuhan University, Wuhan 430072, China

and economic leaps have caused unprecedented environmental degradation (Wu and Wang 2022). Currently, China's rural environmental governance primarily relies on a government-led model, with insufficient participation from market and social entities. This scenario manifests as explicit dependence on administrative measures during market-oriented reforms (Xie and Hao 2024). Additionally, the weak participation awareness among rural residents has led to prevalent "free-riding" behavior, whereby collective members benefit from environmental improvements, regardless of their engagement (Zhang and Guo 2023). A major challenge for rural revitalization lies in addressing these governance gaps (Yang and Pan 2024). Farmers' participation is the key to promoting good governance of the rural environment, but there is a lack of any significant discussion on the effectiveness of farmers' participation in rural environmental governance (Fu et al 2025). Therefore, the integration of "systematic thinking" into the governance framework and guiding residents to participate in environmental policy decision-making is necessary to ensure the strategy's successful implementation.

Previously, scholars have conducted in-depth explorations into the realization pathways of effective rural environmental governance from multiple dimensions, including path analysis, empirical research, and game theory. Qian et al. (2022) employed a Tobit regression model to empirically investigate the factors that influence rural ecological environment governance efficiency in the national context overall and separately in the eastern, central, and western regions of China. The study proposed that the key to improving rural ecological environment governance efficiency in China lies in promoting a differentiated regional coordinated governance mechanism. Zhang et al. (2024) investigated the relationship between China's public environmental issues and rural living environments and explored how public concerns affect living conditions. A fixed-effects model was used to analyze the relationship between public attention and human settlement environments. Based on a mediation model, Du and Jiao (2023) took 285 cities in China, using data from 2017 to 2022 as samples. The study constructed explanatory variables, explained variables, and mediating variables using the entropy method and empirically analyzed the impact of rural infrastructure construction on rural human settlement environment governance and rural economic development. The mediating role of rural human settlement environment governance was also examined. Against the backdrop of the rural revitalization strategy, Zeng et al. (2021) took Shuanglongqiao Village, Fengming Town, Xichong County, Sichuan Province as an empirical research object for perfect countryside planning strategies. The study comprehensively applied research methods, including a literature review, field surveys, and comparative analysis. From the perspective of digital technology enabling rural governance, Chen et al. (2023) used China's land economic survey data to construct an ordered Probit model. That model was used to empirically test the impact of digital governance on the level of domestic waste classification among rural residents. Rural residents' participation in environmental governance is not only an individual behavioral choice but is also influenced by their socioeconomic status. Yang et al. (2025) used the survey data of 2088 rural households in Jiangsu Province, China, to analyze the differential impact of farmers' participation in rural living environment governance. To solve the problem of the uncontrolled consumption of natural resources, which poses a threat to China's rural landscape, Li (2024) proposed a new framework that combines a deep convolutional neural network (CNN) with rural ecological governance. Based on the 2015 China General Social Survey and the statistical data of 89 prefecture-level cities, Liu et al. (2025) constructed an ordered probit model and a mediating effect model. The objective was to explore the logic of environmental regulation and environmental accountability affecting public environmental governance satisfaction. In addition, Qu et al. (2022) reviewed the emerging trends and prospects of wastewater management in China, emphasizing how technological and institutional innovations, including PPP, are reshaping future governance pathways.

Many studies have also been conducted on the game theory of environmental governance actors. Du et al. (2019) employed an evolutionary game model as the theoretical foundation to design multiple symbiotic scenarios. The study conducted an in-depth systematic analysis of the core elements promoting multi-stakeholder collaborative symbiosis in the PPP model for rural environmental governance. Lu and Xu (2023) applied fuzzy-set qualitative comparative analysis (fsQCA) to dissect the influencing factors of rural human settlement environment improvement and those factors' interactive effects. To analyze the behavioral choices of stakeholders in rural environmental third-party governance and provide policy insights for formulating rational solutions, Wu et al. (2023) constructed a game model between local governments and third-party governance agencies. He et al. (2022) used evolutionary game theory to analyze the evolutionary dynamics of farmers' decision-making behavior in rural environmental governance under a crowdfunding model. Liu et al. (2022) introduced key parameters (e.g., local government governance intensity, central government inspection intensity, and government subsidy intensity) into an evolutionary game model. That model was used to analyze the significance of clean heating in northern China's rural areas for environmental governance and carbon neutrality goals. Jiang and Zheng (2024) studied the strategic evolution among village enterprises, governments, and villagers under dynamic reward-punishment mechanisms, finding periodic fluctuations in tripartite strategies without asymptotic stability. The study proposed that governments should raise the penalty ceiling for illegal emissions by village enterprises while simultaneously cautiously increasing the reward ceiling for villagers' proactive supervision.

A PPP (Zhang et al 2023) refers to a collaborative model between governments and private enterprises in which public goods or services are jointly provided. As an emerging approach to public goods provision, the PPP model has been widely promoted in numerous sectors across China in recent years. In the PPP framework (Wang et al 2021, 2024), private enterprises demonstrate superior management expertise and advanced technical capabilities, compared to governments. Incorporating private entities into rural environmental governance can leverage the private entities' managerial and technological advantages. This will help to reduce governance costs, enhance social influence, and broaden financing channels. Feng et al. (2021) argued that the PPP model for rural water environment governance facilitates the attraction of social capital, gives full play to the market's decisive role in resource allocation, coordinates stakeholder interests, and alleviates government budgetary and debt pressures. Liang et al. (2019) applied game theory to explore the practical application of multi-stakeholder PPP projects in environmental governance within China's context. Zhang et al. (2022) conducted in-depth research on the applicability of the PPP model, particularly in the domain of rural environmental governance. Xia et al. (2025) established a tripartite game model of a rural residential environment improvement PPP project with the participation of local government, social capital, and rural enterprises. The study also analyzed the relevant multi-agent collaborative governance mechanism under the influence of various factors. Wen et al. (2024) empirically analyzed the effects of the PPP mode on sewage treatment in China, finding that PPPs can significantly improve treatment performance and reduce environmental risks.

In PPP projects, the actual level of effort invested by private enterprises is regarded as private information. This makes the design of incentive mechanisms to prompt truthful disclosure a key research topic in PPP project governance. To address this issue, principal-agent theory (Zheng et al., 2025; Niu et al 2024) focuses on constructing efficient contractual mechanisms to incentivize agents to act appropriately under conditions of interest conflict and information asymmetry. Within the principal-agent theoretical framework (Wang and Pan 2023; Yuan et al 2021; Wang et al 2022), agents possess specialized "skills and capabilities". As such, they are entrusted with executing specific functions on behalf of principals when contractual relationships are established. The core of principal-agent theory lies in that, when formulating mechanisms to maximize social welfare, principals must consider both incentive compatibility and participation constraints. Specifically, the incentive compatibility constraint requires designing contracts that motivate agents to maximize their expected utility. Meanwhile, the participation constraint mandates that agents' expected utility under the contract must be at least equal to their reservation utility when declining the contract. Ultimately, this means principals must ensure contracts attract agents and align agents' behaviors with principal objectives during design.

At present, rural environment governance under the PPP model mainly focuses on empirical analysis (e.g., in Ref. Zhang et al 2022) and the evolutionary relationship of the interaction among the participating entities (e.g., in Ref. Feng et al 2021; Xia et al 2025). However, existing research on government incentive mechanisms in rural environmental governance is still rather limited. This is especially true with regard to research on the delegation and agency incentive mechanisms of the participating entities in the case of information asymmetry, which is even rarer. Therefore, this paper introduces the PPP financing model into rural environmental governance and employs principal-agent theory to construct an incentive contract model. The aim is to design rational incentive mechanisms to motivate private enterprises to invest actively, ensuring the achievement of environmental governance objectives. Within the PPP framework, local governments and private enterprises establish principal-agent relationships through contracts. Put simply, governments act as principals, and private enterprises serve as agents that typically hold information advantages. Furthermore, to enhance rural residents' sense of participation and responsibility in governance, this study redefines both local governments and rural residents as principals (with private enterprises remaining the agents). An extended PPP incentive contract model is constructed that incorporates resident participation. This approach seeks to complement and address the gaps in existing research.

The main innovation of this paper lies in designing the incentive mechanisms for rural residents' participation and non-participation in the PPP model for rural environmental governance. The results show that the environmental governance model where local governments and rural residents collaborate is more effective. The uncertainty of the external environment and the risk-averse nature of private enterprises will all have an impact on improving the overall efficiency of environmental governance. Based on the model, policy recommendations are proposed.

The remaining part of this article is arranged as follows: The second part constructs the PPP model incentive contract model for rural environmental governance, designs the incentive contract to analyze the incentive effect. The third part considers the participation of rural residents in environmental governance. Under the non-cooperative and cooperative scenarios between rural residents and local governments, it designs the PPP model incentive contract for rural environmental governance and conducts a comparative analysis of the two scenarios. Finally, it presents the conclusion and suggestions of this article.

2 Basic incentive mechanism model

2.1 Model setting

When considering rural environmental governance problems based on the PPP model, the participants are assumed to include the local government and a private enterprise. The local government is the principal and the private enterprise is the agent, and there is an asymmetry of information between the principal and agent. The local government represents the interests of the government and the local rural residents. The government's goal is to achieve the greatest social utility in rural areas, including economic and environmental benefits. The parameters and significance are shown in Table 1.

It is assumed that the private enterprise accepts a task, entrusted to the enterprise by the local government. Then, according to the Holmstrom model (Holmstrom and Milgrom, 1987), the output function of the private enterprise is expressed as:

$$R = qe + \varepsilon \tag{1}$$

where e is the level of effort made by the private enterprise in completing the tasks received from the local government. The expression $q \geq 0$ represents the output coefficient of the marginal benefit of the private enterprise, which is related to the private enterprise's level of effort and which affects its payoff. The variable ε is an exogenous uncertain factor, which is subject to a normal distribution: $\varepsilon \sim N\left(0, \sigma^2\right)$.

According to Ref. (Wang et al 2022), the cost of the private enterprise's efforts is expressed as:

$$C(e) = \frac{\eta e^2}{2} \quad (\eta > 0) \tag{2}$$

where $\eta > 0$ is the cost coefficient when the private enterprise accepts the task from the local government.

Assume that the private enterprise effort level e cannot be observed; however, the results of that effort can be observed. The local government makes decisions independently. Here, the linear function is adopted to reflect the payoff:

$$W = a + bR \tag{3}$$

where the parameter a indicates that the fixed remuneration given by the local government to the private enterprise is independent of output. The variable b is the incentive coefficient of economic benefit brought by the principal to the private enterprise. For each unit of output R, the remuneration of the private enterprise increases by b units.

The local government is risk-neutral, and the risk coefficient is 0.

The private enterprise is risk-averse, and its utility function has the characteristic of having a constant absolute

Table 1 Parameters and their meanings of the basic incentive mechanism model

Parameters	Meanings
e	The level of effort made by the private enterprise in completing the tasks received from the local government
R	Output function of the private enterprise
$q \geq 0$	The output coefficient of the marginal benefit of the private enterprise, which is related to the private enterprise's level of effort and which affects its payoff
$\varepsilon \sim N\left(0, \sigma^2\right)$	An exogenous uncertain factor
$\eta > 0$	The cost coefficient when the private enterprise accepts the task from the local government
a	The fixed remuneration given by the local government to the private enterprise is independent of output
b	The incentive coefficient of the economic benefit brought by the principal to the private enterprise
κ	Absolute risk aversion coefficient of the private enterprise
w	Actual payoff for the private enterprise
$C(e)$	Effort cost of the private enterprise
$\eta_i > 0, i = 1, 2$	The cost coefficient when the private enterprise accepts the separate tasks from the rural residents and the local government, respectively

risk aversion. That is, the expected utility function of the private enterprise is $u_E = -e^{-\kappa w}$, where parameter w is the actual payoff for the private enterprise, and κ is the absolute risk aversion coefficient of the private enterprise. Assuming that the effort cost $C(e)$ of the private enterprise is equivalent to the monetary cost, the actual benefit of the private enterprise is expressed as:

$$\begin{aligned} &w(R) = W(R) - C(e) = \\ &a + bR - \frac{\eta e^2}{2} = a + b(qe + \varepsilon) - \frac{\eta e^2}{2}. \end{aligned} \tag{4}$$

Thus, the deterministic benefit for the private enterprise is:

$$\begin{aligned} CE &= E(w) - \frac{1}{2}\kappa b^2\sigma^2 \\ &= a + bR - \frac{\eta e^2}{2} - \frac{1}{2}\kappa b^2\sigma^2 \\ &= a + bqe - \frac{\eta e^2}{2} - \frac{1}{2}\kappa b^2\sigma^2 \end{aligned} \tag{5}$$

where $E(w)$ refers to the expected benefits of the private enterprise, and $\frac{1}{2}\kappa b^2\sigma^2$ refers to the risk costs of the private enterprise. In other words, the private enterprise prefers to give up $\frac{1}{2}\kappa b^2\sigma^2$ of the expected gains in exchange for deterministic gains. When $b = 0$, the risk cost is 0. In this case, the private enterprise's maximized expected utility function u_E is equivalent to maximizing the deterministic return from Eq. (5) above.

2.2 Modeling

When the private enterprise acts as the agent of the local government, the local government designs an optimal incentive mechanism, based on its own interests. The contract sequence between the local government and the private enterprise in the presence of a moral hazard is shown in Fig. 1.

Assuming that the total reward of the private enterprise is $U(w)$, the private enterprise will choose the optimal contract according to the incentive mechanism of the local government. This results in a function that maximizes utility as follows:

$$\begin{aligned} &\max_e E[U(w)] \\ &= \max_e E\{U[W(R) - C(e)]\} \\ &= \max_e E(CE) \end{aligned} \tag{6}$$

By integrating Eqs. (1–5) embedded in the assumptions above into the utility maximization function, one can obtain:

$$\max_e E[U(w)] = \max_e \left(a + qbe - \frac{\eta}{2}e^2\right) \tag{7}$$

Because private enterprises have an advantage in terms of information possession, local governments lack the means to directly monitor the actual efforts made by the enterprises. This information asymmetry makes it difficult for local governments to accurately assess the effort level of private enterprises when performing tasks. This issue may affect the effective design and implementation of incentive mechanisms. Therefore, when designing a linear incentive contract, the local government (as the client) needs to deal with two key constraints, namely the participation constraint (IR) and the incentive compatibility constraint (IC).

Then, the incentive contract model (Model 8) under the principal-agent model constructed by local governments and private enterprises in the field of rural environmental governance is:

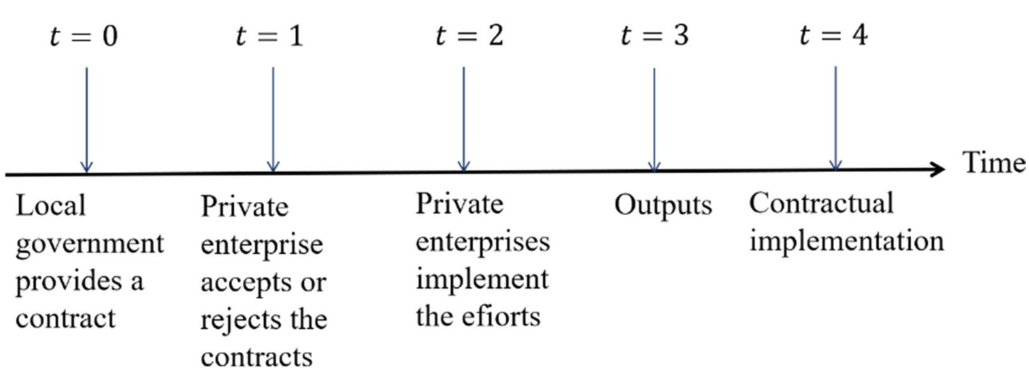

Fig. 1 Contractual timing of local government and private enterprise under moral hazard

$$\max_b E(u) = (1-b)qe - a$$

s.t.

$$\text{(IR)} \quad a + bqe - \frac{\eta e^2}{2} - \frac{1}{2}\kappa b^2\sigma^2 \ge \overline{U}$$

$$\text{(IC)} \quad e^* \in \arg\max_e \left(a + bqe - \frac{\eta e^2}{2} - \frac{1}{2}\kappa b^2\sigma^2\right) \tag{8}$$

In Model 8, $\overline{U}$ represents the retained utility level of the private enterprises. Private enterprises will only agree to accept the incentive contract proposed by the local government when the benefits provided exceed their retained utility. This means that, before accepting the contract, private enterprises will compare the incentives provided by the local governments with their own retained utility. This will be done to ensure that participating in environmental governance projects can bring the private enterprises sufficient benefits.

2.3 Incentive effect analysis

In the process of formulating the incentive contract, the first step is to determine the optimal effort level of the private enterprise. Subsequently, the linear contract is used to determine the optimal incentive contract of the principal, thereby motivating the agent to exert the optimal effort level. Based on this, Model I is solved to obtain Proposition 1.

1 Proposition 1.

The optimal incentive contract of the local government is:

$$b^* = \frac{q^2}{q^2 + \kappa\eta\sigma^2} \tag{9}$$

1 *Proof:*

From Eq. (7), one can obtain the incentive compatibility constraint (IC) using the first order derivative:

$$e^* = \frac{qb}{\eta}. \tag{10}$$

At the same time, the participation constraint of the private enterprise (IR) is:

$$\max_e E(CE) \ge \overline{U} \tag{11}$$

where $\overline{U}$ is the reserve payoff level (i.e., the level of the outside opportunity) of the private enterprise. For the private enterprise to accept the incentive contract, the benefit of the contract designed by the local government must be greater than $\overline{U}$. Therefore, the local government must meet both IC and IR conditions for the private enterprise to engage and fulfill the tasks from the government. According to Eq. (11), one can obtain:

$$a + bqe - \frac{\eta e^2}{2} - \frac{1}{2}\kappa\eta^2\sigma^2 \ge \overline{U}. \tag{12}$$

The local government is risk-neutral, and its expected utility is equal to the expected payoff. This is expressed as:

$$\max_b E[U(R-a-bR)] = \max_b E(R-a-bR). \tag{13}$$

By introducing the IC and the IR conditions into Eq. (13), the following equations are generated:

$$\begin{aligned} &\max_b E[U(R-a-bR)] \\ &= \max_b E\left[U\left(qe - a - bqe - \frac{1}{2}\kappa\eta^2\sigma^2\right)\right] \\ &= qe - a - bqe - \frac{1}{2}\kappa\eta^2\sigma^2. \end{aligned} \tag{14}$$

The optimal incentive contract for the local government in Eq. (9) is obtained by solving the first derivative of b in Eq. (14).

Using Proposition 1 in Sect. 2 yields Conclusion 1.

Conclusion 1 For the private enterprise, a higher level of risk aversion is associated with greater variances in output level, smaller incentives, and more concerns about investing hard work. The greater the output coefficient of marginal benefits and the greater the incentive provided to the private enterprise are, the harder the private enterprise works.

Conclusion 1 indicates that the optimal incentive contract in Eq. (9) assumes that the private enterprise must bear certain risks for the process of environmental governance. In particular, the incentive coefficient of the economic benefit brought by the principal to the private enterprise is the decreasing function of the absolute risk aversion coefficient of the private enterprise and exogenous uncertain factor. In other words, the higher the degree of risk aversion of the private enterprise is and the greater the variance of output is, the more the private enterprise is afraid to work hard, and the less risk the enterprise should bear. In addition, the incentive coefficient is the increasing function of the output coefficient of the marginal benefit of the private enterprise expressed in Eq. (9). In other words, the greater the output coefficient of marginal benefit is, the harder the private enterprise is expected to work.

3 Incentive mechanism model considering rural residents' participation

Section 2 established the PPP model incentive mechanism model of rural environmental governance between the local government and a private enterprise. However, rural environmental protection involves the local government and rural residents. In addition, successful experiences in rural environmental governance demonstrate the need to mobilize the rural residents. This means relying on rural residents and perfecting the guiding mechanisms of their participation. This highlights the advantages of having the local government, a private enterprise, and rural residents implement multi-agent joint governance. Doing so helps effectively address the outstanding problems of rural environmental pollution control and advances rural environmental improvement and rural revitalization.

It may be advisable for China's environmental rule of law to move away from the two-dimensional relationship between local government and private enterprise. Instead, a three-tier principal-agent model should be constructed involving the local government, a private enterprise, and rural residents. To assess this proposed change, in Sect. 3, a rural environmental governance incentive mechanism model is established that considers the participation of rural residents. The local government and rural residents are the two principals, and the private enterprise is the agent. The principal-agent relationship in the presence of a moral hazard is shown in Fig. 2, and the contract timing is shown in Fig. 3.

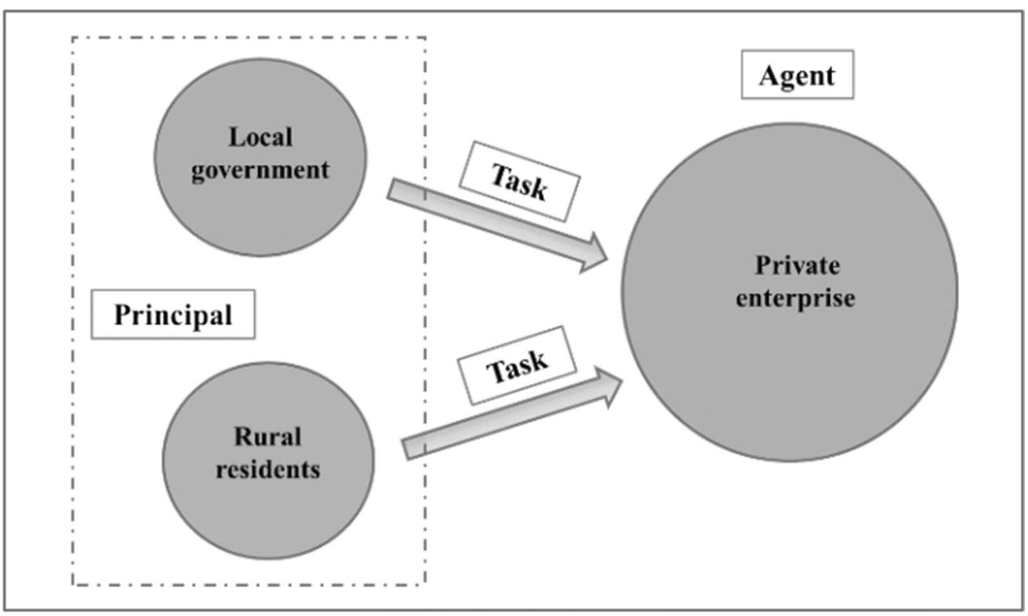

Fig. 2 Principal-agent relationship between the local government, a private enterprise, and rural residents

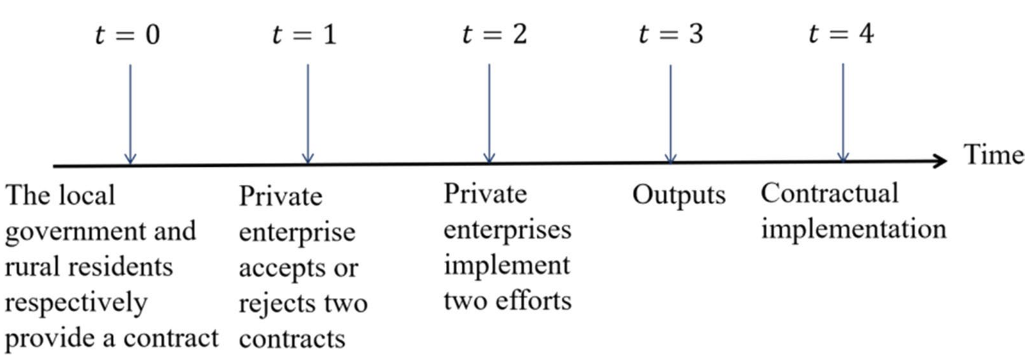

Fig. 3 Moral hazard contractual timing between the local government, rural residents, and a private enterprise

The following section discusses different scenarios that consider different levels of cooperation between the government and resident principals. The parameters and significance of this section are shown in Table 2.

3.1 Model of local government and rural residents with no cooperation

3.1.1 Model

Assume the participants in the principal-agent problem include rural residents, the local government, and a private enterprise. Rural residents and the local government are the two principals, and the private enterprise is the agent. The agent accepts two tasks from the two different principals. Information asymmetry is assumed to exist between the participants. Assume the local government represents rural interests and rural residents. The function of the local government is to achieve economic and environmental benefits, including the greatest overall social utility. The rural residents want to obtain the greatest economic and environmental benefits.

When rural residents and the local government do not cooperate, the private enterprise is entrusted with tasks from both the rural residents and the local government, as two separate principals. According to the Holmstrom model (Zhang et al 2022), the expected benefits for the private enterprise for each task are:

$$R_i = q_i e_i + \varepsilon_i, \quad i = 1, 2 \tag{15}$$

where $i = 1$ represents rural residents, and $i = 2$ represents the local government. The expression $e_i, i = 1, 2$ represents the effort of the private enterprise to complete the entrusted task given by principal i when the principals do not cooperate. The efforts $e_i, i = 1, 2$ are independent. The expression $q_i \geq 0, i = 1, 2$ represents the output coefficient of the marginal benefit of the private enterprise, which is related to the level of effort from the private enterprise and affects the enterprise's payoff for each task. The variable $\varepsilon_i, i = 1, 2$ is an exogenous uncertain factor, which is subject to normal distribution: $\varepsilon_i \sim N(0, \sigma^2)$.

The cost of the efforts made by the private enterprise is expressed as:

$$C(e_1, e_2) = \frac{\eta_1 e_1^2}{2} + \frac{\eta_2 e_2^2}{2} (\eta_i > 0, i = 1, 2) \tag{16}$$

where $\eta_i > 0, i = 1, 2$ represents the cost coefficient when the private enterprise accepts the separate tasks from the rural residents and the local government, respectively.

Assume that the effort $e_i, i = 1, 2$ implemented by the private enterprise cannot be observed, but the results of that effort can be observed. Rural residents and the local government make decisions independently, in the form of the linear function payoff:

$$W_i = a_i + b_{ii} R_i + b_{ji} R_j, i = 1, 2, i \neq j \tag{17}$$

where $a_i, i = 1, 2$ is the fixed payment given by the principal to the private enterprise, which has nothing to do with output, and b_{ii} is the incentive coefficient of the economic benefits provided by the principal to the private enterprise. In other words, for every unit increase in output R_i, the remuneration paid to the private enterprise will increase by b_{ii}. The variable b_{ij} is the incentive coefficient of the environmental benefits provided to principal j by principal i. The external effects obtained by the private enterprise when completing the two tasks include the financial subsidies and rewards given by the local government and the additional effects created for the private enterprise by rural residents. This includes improvements to the enterprise's positive image and earnings.

Table 2 Parameters and their meanings of the model considering rural residents' participation

Parameters	Meanings
$\eta_i > 0, i = 1, 2$	The cost coefficient when the private enterprise accepts the separate tasks from the rural residents and the local government, respectively
$a_i, i = 1, 2$	Fixed payment given by the principal to the private enterprise
$R_i, i = 1, 2$	Expected benefits to the private enterprise for each task
$q_i \geq 0, i = 1, 2$	Output coefficient of the marginal benefit of the private enterprise
$\varepsilon_i \sim N(0, \sigma^2)$	Exogenous uncertain factor
$e_i, i = 1, 2$	The effort of the private enterprise to complete the entrusted task given by rural residents and the local government
$b_{ij}, i, j = 1, 2$	The incentive coefficient of the economic benefits provided by the principal to the private enterprise
w'	Actual payoff to the private enterprise when rural residents and the local government do not cooperate
$\xi \sim N(0, \sigma^2)$	The uncertainty of the external environment
$\zeta(1 - \zeta)$	The importance of the economic and environmental benefits for the private enterprise, respectively
a_C	Fixed remuneration of the private enterprise when the local government and rural residents cooperate
b_C	The incentive coefficient brought to the private enterprise by the local government and rural residents as co-trustees

Rural residents and local government are assumed to be risk-neutral, with a risk coefficient of 0. Private enterprises are risk-averse, and their utility functions are characterized by constant absolute risk-averse, i.e., $u_E' = -e^{-\kappa w'}$. In this expression, w' is the actual payoff to the private enterprise when rural residents and the local government do not cooperate, and κ is the absolute risk aversion coefficient of the private enterprise. When the private enterprise acts on behalf of the rural residents and the local government through the two separate tasks, the enterprise's actual payoff is $W_1(R_1) + W_2(R_2)$. The definite equivalent payoff to the private enterprise is:

$$\begin{aligned} CE &= E(w') - \frac{1}{2}\kappa\sigma^2\left[(b_{11}+b_{12})^2 + (b_{21}+b_{22})^2\right] \\ &= E[W_1(R_1)] + E[W_2(R_2)] - C(e_1,e_2) - \frac{1}{2}\kappa\sigma^2\left[(b_{11}+b_{12})^2 + (b_{21}+b_{22})^2\right] \\ &= (a_1+a_2) + q_1(b_{11}+b_{12})e_1 + q_2(b_{21}+b_{22})e_2 - \left[\frac{\eta_1}{2}e_1^2 + \frac{\eta_2}{2}e_2^2\right] \\ &\quad -\frac{\kappa\sigma^2}{2}\left[(b_{11}+b_{12})^2 + (b_{21}+b_{22})^2\right] \end{aligned} \tag{18}$$

where $\frac{1}{2}\kappa\sigma^2\left[(b_{11}+b_{12})^2 + (b_{21}+b_{22})^2\right]$ is the cost of risk to the private enterprise. This represents the cost at which the private enterprise prefers to give up the payoff $\frac{1}{2}\kappa\sigma^2\left[(b_{11}+b_{12})^2 + (b_{21}+b_{22})^2\right]$ in the expected payoff $E(w')$, to receive the more certain payoff. When $b_{ij} = 0, i,j = 1,2$, the cost of risk is 0. The maximized expected utility function u_E' of the private enterprise is equivalent to the maximization of the above deterministic payoff generated in Eq. (18).

When the private enterprise acts as an agent for both rural residents and the local government for two different tasks, the total payment to the private enterprise is $W_1(R_1) + W_2(R_2)$. Rural residents and the local government will design incentive mechanisms based on their own interests. The private enterprise will then choose the optimal combination, according to the incentive mechanism, yielding the function of utility maximization:

$$\begin{aligned} &\max_{e_1e_2} E[U(w')] \\ &= \max_{e_1e_2} E\{U[W_1(R_1) + W_2(R_2) - C(e_1,e_2)]\} \\ &= \max_{e_1e_2} E(CE) \end{aligned} \tag{19}$$

According to the assumptions in Sect. 3, the utility maximization function is:

$$\begin{aligned} &\max_{e_1e_2} E[U(w')] \\ &= \max_{e_1e_2}\left\{\begin{array}{l} (a_1+a_2) + q_1(b_{11}+b_{12})e_1 + q_2(b_{21}+b_{22})e_2 \\ -\left[\frac{\eta_1}{2}e_1^2 + \frac{\eta_2}{2}e_2^2\right] - \frac{\kappa\sigma^2}{2}[(b_{11}+b_{12})^2 + (b_{21}+b_{22})^2] \end{array}\right\}. \end{aligned} \tag{20}$$

Based on the above model description, an incentive contract model (Model 21) between local governments and rural residents in a non-cooperative state was constructed, as follows:

$$\begin{aligned} &\max_{(a_i,b_{ij}),i,j=1,2} E(u) = (1-b_{11}-b_{12})q_1e_1 + (1-b_{21}-b_{22}) \\ &q_2e_2 - a_1 - a_2 \text{s.t.} \\ &(\text{IR})\ (a_1+a_2) + q_1(b_{11}+b_{12})e_1 + q_2(b_{21}+b_{22})e_2 \\ &-\left(\frac{\eta_1}{2}e_1^2 + \frac{\eta_2}{2}e_2^2\right) - \frac{\kappa\sigma^2}{2}[(b_{11}+b_{12})^2 + (b_{21}+b_{22})^2] \ge \overline{U} \\ &(\text{IC})\ e_1^*, e_2^* \\ &\in \arg\max_{e_1,e_2}\left\{\begin{array}{l} (a_1+a_2) + q_1(b_{11}+b_{12})e_1 + q_2(b_{21}+b_{22})e_2 \\ -\left(\frac{\eta_1}{2}e_1^2 + \frac{\eta_2}{2}e_2^2\right) \\ -\frac{\kappa\sigma^2}{2}[(b_{11}+b_{12})^2 + (b_{21}+b_{22})^2] \end{array}\right\} \end{aligned} \tag{21}$$

3.1.2 Incentive effect analysis

1 Proposition 2.

When the local government and rural residents do not cooperate, the optimal incentive contract for rural residents is:

$$\begin{cases} \dfrac{q_1^2}{\eta_1} - \dfrac{(q_1^2 + \eta_1\kappa\sigma^2)(b_{11}+b_{12})}{\eta_1} + \dfrac{q_1^2 b_{12}}{\eta_1} = 0 \\ -\dfrac{(q_2^2 + \eta_2\kappa\sigma^2)(b_{21}+b_{22})}{\eta_2} + \dfrac{q_2^2 b_{22}}{\eta_2} = 0 \end{cases}. \tag{22}$$

The optimal incentive contract for the local government is:

$$\begin{cases} -\dfrac{(q_1^2 + \eta_1\kappa\sigma^2)(b_{11}+b_{12})}{\eta_1} + \dfrac{q_1^2 b_{11}}{\eta_1} = 0 \\ \dfrac{q_2^2}{\eta_2} - \dfrac{(q_2^2 + \eta_2\kappa\sigma^2)(b_{21}+b_{22})}{\eta_2} + \dfrac{q_2^2 b_{21}}{\eta_2} = 0 \end{cases}. \tag{23}$$

1 *Proof:*

By calculating the first derivative of Eq. (20) with respect to $e_i, i = 1,2$, the two respective incentive compatibility constraint (IC) values are calculated:

$$e_1 = \frac{q_1(b_{11}+b_{12})}{\eta_1}, e_2 = \frac{q_2(b_{21}+b_{22})}{\eta_2}. \tag{24}$$

The participation constraint (or individual rationality constraint) (IR) of the private enterprise is:

$$\max_{e_1e_2} E(CE) \geq \overline{U}_1 \tag{25}$$

where, $\overline{U}$ is the reserve payoff level of the private enterprise. For the private enterprise to accept the incentive contract, the benefit of the contract designed by the rural residents and the local government must be greater than the private enterprise $\overline{U}$. Therefore, rural residents and the local government must meet both IC and IR conditions for the private enterprise to accept the contract and complete agency tasks.

According to Assumption 3.6, rural residents are risk-neutral, so the rural residents' expected utility is equal to the expected payoff. Therefore:

$$\begin{aligned}&\max_{b_{11}b_{21}} E[U_1(R_1 - a_1 - b_{11}R_1 - b_{21}R_2)]\\&= \max_{b_{11}b_{21}} E(R_1 - a_1 - b_{11}R_1 - b_{21}R_2)\end{aligned} \tag{26}$$

Substituting the incentive compatible constraint (IC) and participating constraint (IR) into Eq. (26) yields:

$$\begin{aligned}&\max_{b_{11}b_{21}} E[U_1(R_1 - a_1 - b_{11}R_1 - b_{21}R_2)]\\&= \max_{b_{11}b_{21}} \left\{ \begin{array}{l} \frac{q_1^2(b_{11}+b_{12})}{\eta_1} - \frac{(q_1^2+\eta_1\kappa\sigma^2)(b_{11}+b_{12})^2}{2\eta_1} - \frac{(q_2^2+\eta_2\varphi\sigma^2)(b_{21}+b_{22})^2}{2\eta_2} \\ + \frac{q_1^2 b_{12}(b_{11}+b_{12})}{\eta_1} + \frac{q_2^2 b_{22}(b_{21}+b_{22})}{\eta_2} + a_2 - \overline{U}_1 \end{array} \right\}\end{aligned} \tag{27}$$

The first partial derivative of Eq. (27) is calculated, and the optimal incentive contract for rural residents is generated using Eq. (22).

In addition, based on Assumption 3.4, the local government is also risk-neutral, and the government's expected utility is equal to its expected payoff. Therefore, the expected utility is:

$$\max_{b_{22}b_{12}} E[U_2(R_2 - a_2 - b_{22}R_2 - b_{12}R_1)]] = \max_{b_{22}b_{12}} E(R_2 - a_2 - b_{22}R_2 - b_{12}R_1). \tag{28}$$

Substituting the conditions (IC) and (IR) into Eq. (10) yields:

$$\begin{aligned}&\max_{b_{22}b_{12}} E[U_2(R_2 - a_2 - b_{22}R_2 - b_{12}R_1)]\\&= \max_{b_{22}b_{12}} \left\{ \begin{array}{l} \frac{q_2^2(b_{21}+b_{22})}{\eta_2} + \frac{q_1^2 b_{11}(b_{11}+b_{12})}{\eta_1} + \frac{q_2^2 b_{21}(b_{21}+b_{22})}{\eta_2} - \\ \frac{(q_1^2+\eta_1\kappa\sigma^2)(b_{11}+b_{12})^2}{2\eta_1} - \frac{(q_2^2+\eta_2\kappa\sigma^2)(b_{21}+b_{22})^2}{2\eta_2} + a_1 - \overline{U} \end{array} \right\}.\end{aligned} \tag{29}$$

The first partial derivative of Eq. (27) is solved. Then, the optimal incentive contract for rural residents is obtained using Eq. (23).

The model settings in Sect. 3.1.2 lead to Conclusions 2 and 3:

Conclusion 2 When the local government and rural residents, as two principals, individually give a private enterprise a task, the conflicting incentives can lead to negative impacts between local governments and rural residents. Such impacts may weaken the incentive for the private enterprise. When the local government and rural residents both increase incentives, the private enterprise does not increase the level of effort.

Adding Eq. (22) and Eq. (23) yields:

$$b_{11} + b_{12} = \frac{q_1^2}{q_1^2 + 2\eta_1\kappa\sigma^2} \tag{30}$$

Substituting Eq. (30) into Eq. (22) yields $b_{12} < 0$. Similarly, $b_{21} < 0$.

The incentive coefficient for private enterprise in terms of earnings is negative. At the same time, because the level of effort for the private enterprises is determined, there are constraints:

$$e_1 + e_2 \leq e \tag{31}$$

In other words, when the local government increases incentives, the private enterprise may reduce the level of effort devoted to the task given by rural residents. The reverse may also occur. Even though local governments and rural residents have increased the incentives to the private enterprise, the private enterprise does not improve its efforts. When the local government and rural residents do not cooperate, the interests of both principals may be negatively affected, decreasing the good governance effect.

3.2 Model of local government and rural residents in cooperation

Based on the analysis in Sect. 3.1, this section analyzes the overall interests of both principals and establishes an incentive mechanism model for the private enterprise in the case in which the local government and rural residents cooperate.

3.2.1 Complementary assumptions

Using the Cobb–Douglas production function (C-D function), the total payoff of the local government and rural residents is expressed as:

$$R_C = f(e_1, e_2) + \xi = \gamma e_1^{\zeta} e_2^{1-\zeta}, \quad 0 < \gamma < 1 \tag{32}$$

The variable $\xi \sim N(0, \sigma^2)$ represents the uncertainty of the external environment; ζ and $1-\zeta$ represent the importance of the economic and environmental benefits for the private enterprise, respectively. The marginal outputs of the private enterprise to the two principals are $\gamma\zeta e_1^{\zeta-1} e_2^{1-\zeta}$ and $\gamma(1-\zeta)e_1^{\zeta} e_2^{-\zeta}$, respectively. When the local government and rural residents cooperate, the marginal output of a private enterprise completing one task is an increasing function of another task, and γ represents the comprehensive capacity level of the private enterprise.

The payment for the private enterprise adopts a linear function:

$$W_C = a_C + b_C R_C \tag{33}$$

where a_C is the fixed remuneration of the private enterprise when the local government and rural residents cooperate, also known as the normal payoff; b_C is the incentive coefficient brought to the private enterprise by the local government and rural residents as co-trustees. In other words, the remuneration of the private enterprise increases by b_C for every unit of the total payoff.

3.2.2 Model

According to the above complementary assumptions, the deterministic payoff for the private enterprise is:

$$\begin{aligned} \max_{e_1 e_2} CE &= E(W_C) - C(e_1, e_2) - \frac{1}{2}\kappa b_C^2 \sigma^2 \\ &= a_C + b_C \gamma e_1^{\zeta} e_2^{1-\zeta} - \frac{1}{2}\eta_1 e_1^2 - \frac{1}{2}\eta_2 e_2^2 - \frac{1}{2}\kappa b_C^2 \sigma^2 \end{aligned} \tag{34}$$

The assumptions in Sect. 3 and Eq. (34) yield Proposition 3, as follows:

1 Proposition 3.

Given the above complementary assumptions, the joint optimal incentive coefficient of the local government and rural residents is:

$$b_C^* = \frac{\gamma^2 \left(\frac{\zeta}{\eta_1}\right)^{\zeta} \left(\frac{1-\zeta}{\eta_2}\right)^{1-\zeta}}{\gamma^2 \left(\frac{\zeta}{\eta_1}\right)^{\zeta} \left(\frac{1-\zeta}{\eta_2}\right)^{1-\zeta} + \kappa\sigma^2} \tag{35}$$

1 *Proof:*

The first derivative of Eq. (36) is used to generate the incentive compatibility constraint (IC):

$$\begin{aligned} e_1 &= \left[\frac{\zeta\eta_2}{(1-\zeta)\eta_1}\right]^{\frac{1}{2}} \gamma \left[\frac{\zeta}{\eta_1}\right]^{\frac{\zeta}{2}} \left[\frac{1-\zeta}{\eta_2}\right]^{\left(1-\frac{\zeta}{2}\right)} \\ b_C &= \gamma b_C \left[\frac{\zeta}{\eta_1}\right]^{\frac{1+\zeta}{2}} \left[\frac{1-\zeta}{\eta_2}\right]^{\left(\frac{1-\zeta}{2}\right)} \\ e_2 &= \gamma b_C \left[\frac{\zeta}{\eta_1}\right]^{\frac{\zeta}{2}} \left[\frac{1-\zeta}{\eta_2}\right]^{\left(1-\frac{\zeta}{2}\right)} \end{aligned} \tag{36}$$

The utility of the local government and rural residents when they cooperate is:

$$\pi = R_C - W_C \tag{37}$$

The expected utility is:

$$E(\pi) = E(R_C - W_C) \tag{38}$$

Therefore:

$$\begin{aligned} \max_{a_C, b_C} E(\pi) &= \max_{a_C, b_C} E(R_C - W_C) \\ &= \max_{a_C, b_C} \left[\gamma e_1^{\zeta} e_2^{1-\zeta} - E(W_C)\right] \end{aligned} \tag{39}$$

The participating constraint condition (IR) is:

$$E(W_C) - \frac{1}{2}\eta_1 e_1^2 - \frac{1}{2}\eta_2 e_2^2 - \frac{1}{2}\kappa b_C^2 \sigma^2 \geq \overline{U}_C \tag{40}$$

where $\overline{U}_C$ is the retained payoff level of the private enterprise when the local government cooperates with rural residents. Substituting the IC of Eq. (34) and the IR of Eq. (40) into Eq. (39) yields:

$$\begin{aligned}\max_{a_C,b_C} E(\pi) &= \left\{\gamma\left[\gamma b_C^2\left(\frac{\zeta}{\eta_1}\right)^{\frac{1+\zeta}{2}}\left(\frac{1-\zeta}{\eta_2}\right)^{\left(\frac{1-\zeta}{2}\right)}\right]^{\zeta}\left[\gamma b_C^2\left(\frac{\zeta}{\eta_1}\right)^{\frac{\zeta}{2}}\left(\frac{1-\zeta}{\eta_2}\right)^{\left(1-\frac{\zeta}{2}\right)}\right]^{1-\zeta} - \overline{U}_C - \frac{1}{2}\eta_1 e_1^2 - \frac{1}{2}\eta_2 e_2^2 - \frac{1}{2}\kappa b_C^2\sigma^2\right\} \\ &= \max_{a_C,b_C}\left[\gamma^2 b_C\left(\frac{\zeta}{\eta_1}\right)^{\zeta}\left(\frac{1-\zeta}{\eta_2}\right)^{1-\zeta} - \frac{1}{2}\eta_1\gamma^2 b_C^2\left(\frac{\zeta}{\eta_1}\right)^{1+\zeta}\left(\frac{1-\zeta}{\eta_2}\right)^{1-\zeta} - \frac{1}{2}\eta_2\gamma^2 b_C^2\left(\frac{\zeta}{\eta_1}\right)^{\zeta}\left(\frac{1-\zeta}{\eta_2}\right)^{(2-\zeta)} - \frac{1}{2}\kappa b_C^2\sigma^2 - \overline{U}_C\right]\end{aligned} \tag{41}$$

By taking the first partial derivative of Eq. (41) with respect to b_C, the optimal incentive coefficient is expressed as:

$$b_C^* = \frac{\gamma^2\left(\frac{\zeta}{\eta_1}\right)^{\zeta}\left(\frac{1-\zeta}{\eta_2}\right)^{1-\zeta}}{\eta_1\gamma^2\left(\frac{\zeta}{\eta_1}\right)^{1+\zeta}\left(\frac{1-\zeta}{\eta_2}\right)^{1-\varsigma} + \eta_2\gamma^2\left(\frac{\zeta}{\eta_1}\right)^{\zeta}\left(\frac{1-\zeta}{\eta_2}\right)^{(2-\zeta)} + \kappa\sigma^2} \tag{42}$$

After simplifying Eq. (42), the optimal incentive coefficient is generated using Eq. (35).

Equation (35) shows that the optimal incentive coefficient of b_C^* is affected by parameter κ, σ^2, γ, η_1, η_2 and ζ. Therefore, Sect. 3.2.2 analyzes the relationship between the optimal incentive coefficient b_C^* of the model in the cooperative state and with different parameters.

3.2.3 Incentive effect analysis

Using the analytical results in Sect. 3.2.2, this study numerically simulates the relationship between the optimal excitation coefficients b_C^* and $\kappa, \sigma^2, \gamma, \eta_1, \eta_2, \zeta$ of the model in a cooperative state.

(1) The impact of the degree of risk aversion κ and variance σ^2 of uncertainty in the external environment on the incentive coefficient b_C^*.

Conclusion 3 With increases in the degree of risk aversion κ and the variance σ^2 in the uncertainty in the external environment, there is a decrease in the incentive coefficient b_C^* of the private enterprise. In other words, the higher the risk aversion of private enterprise and the greater the uncertainty in the external environment are, the lower the incentive for the private enterprise is expected to be.

Figure 4 (left panel) and the first partial derivative of b_C^* with respect to these two parameters show that, as the degree of risk aversion κ and variance σ^2 of external environment uncertainty increase, the incentive coefficient b_C^* of the private enterprise decreases. This indicates that, when a private enterprise has a higher degree of risk aversion and the external environment is more uncertain, the incentive for the private enterprise is expected to decline. When designing the incentive contract, the principal considers the private enterprise's degree of risk aversion and the uncertainty of the external environment. The principal adopts differentiated incentives accordingly. A

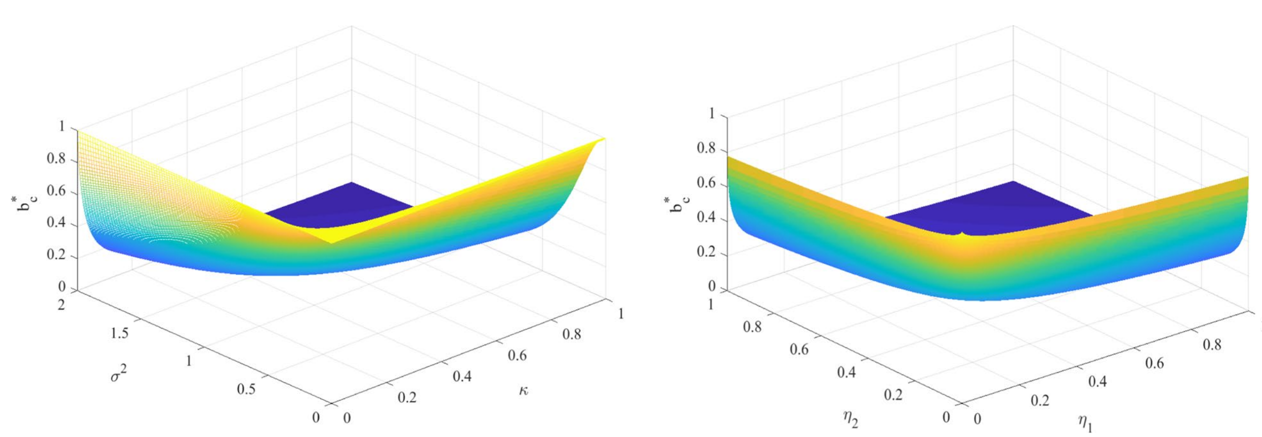

Fig. 4 The impact of the degree of risk aversion coefficient κ and variance σ^2 of external environment uncertainty on the incentive coefficient b_C^*, where $\gamma = 1, \zeta = 0.5, \eta_1 = 5$, and $\eta_2 = 5$(left panel), and the influence of the private enterprise's cost coefficient η_1, η_2 on incentive coefficient b_C^*, where $\gamma = 1, \kappa = 0.5, \zeta = 0.5$, and $\sigma^2 = 9$ (right panel)

private enterprise with low capital strength, less experience, and an unfavorable external environment tends to be risk-averse and may reduce incentives to pursue deterministic returns. A private enterprise with a strong capital capacity that can accommodate challenges and has high enthusiasm and a high risk tolerance can increase incentives. This may mobilize the private enterprise to take the initiative to invest effort. In addition, the local government should implement measures to reduce the uncertainty of the external environment, such as perfecting policies and providing oversight. These actions may create a healthier external environment.

(2) The influence of the private enterprise's cost coefficient η_1, η_2 on incentive coefficient b_c^*.

Conclusion 4 An increase in the cost coefficient of the private enterprise weakens the incentive effect for that enterprise.

Figure 4 (right panel) shows that, when η_1 is constant, then $\frac{\partial b_C^*}{\partial \eta_2} < 0$. As such, the surface in the figure shows a downward trend. When η_2 is constant, then $\frac{\partial b_C^*}{\partial \eta_1} < 0$. As such, the surface in the figure again shows a downward trend. This indicates that the optimal incentive coefficient b_C^* of the private enterprise is inversely proportional to the cost coefficient. In other words, an increase in the cost coefficient will weaken the incentive effect for the private enterprise. Therefore, if the cost coefficient for the private enterprise when accepting the tasks from rural residents and the local government is too high, the private enterprise's incentive will be weakened. In contrast, a private enterprise with a lower cost coefficient is likely to be more willing to improve the environment, appropriately increasing the incentive.

(3) The influence of the comprehensive ability level γ of the private enterprise on incentive coefficient b_c^*.

Conclusion 5 As the private enterprise's comprehensive ability level γ improves, the optimal incentive coefficient b_C^* of the private enterprise increases.

Figure 5 (left panel) shows that, as the comprehensive ability level γ of the private enterprise increases, the optimal incentive coefficient b_C^* of the private enterprise increases. In other words, $\frac{\partial b_C^*}{\partial \gamma} > 0$. The principal should consider the ability level of the private enterprise when designing the incentive contract and provide differentiated incentives accordingly. Private enterprises with high capacity and advantages in capital and information may be more willing to invest effort in the process of environmental governance. The principal should therefore increase incentives to create better environmental benefits.

(4) The influence of economic benefit and environmental benefit importance on incentive coefficient b_C^*.

Let $\psi = \left[\frac{\zeta}{\eta_1}\right]^{\zeta}\left[\frac{1-\zeta}{\eta_2}\right]^{1-\zeta}\gamma^2$. Then, Eq. (42) can be expressed as $b_C^* = \frac{\psi}{\psi+\kappa\sigma^2}$. According to $\frac{\partial \psi}{\partial \zeta}$, the relationship between ψ and ζ is influenced by the output coefficient γ and the cost coefficient η_1, η_2 of the private enterprise. Let the private enterprise's input–output ratios in terms of economic and environmental effects be $\tau = \frac{\zeta}{\eta_1}$ and $\upsilon = \frac{1-\zeta}{\eta_2}$, respectively. In that case, $\frac{\partial \psi}{\partial \zeta} = \psi \ln\left[\frac{\tau}{\upsilon}\right]$. In addition, $\frac{\partial^2 \psi}{\partial \zeta^2} = \ln^2\left[\frac{\tau}{\upsilon}\right] + \frac{\psi}{1+\zeta} > 0$. As such, the second derivative is convex. When $\tau > \upsilon$, then $\frac{\partial \psi}{\partial \zeta} > 0$. This shows that, when the input–output

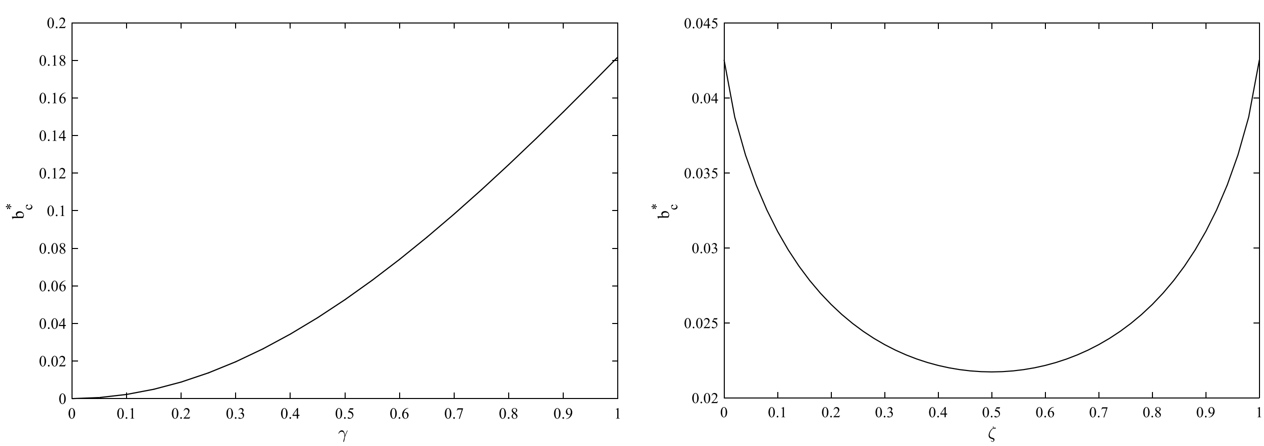

Fig. 5 The influence of the comprehensive ability level γ of the private enterprise on incentive coefficient b_C^*, where $\kappa = 0.5, \varsigma = 0.5, \eta_1 = 5$, $\eta_2 = 5$, and $\sigma^2 = 9$ (left panel). The influence of the economic benefit level ζ of the private enterprise on incentive coefficient b_C^*, where $\kappa = 0.5$, $\gamma = 1, \eta_1 = 5, \eta_2 = 5$, and $\sigma^2 = 9$ (right panel)

efficiency of the private enterprise is higher, the optimal incentive coefficient b_C^* is an increasing function of ζ. Similarly, when the input–output efficiency of environmental benefits is higher, the optimal incentive coefficient b_C^* is a subtractive function of ζ. In addition, a numerical analysis yields Fig. 5 (right panel), which yields the same results. The above analysis leads to Conclusion 6.

Conclusion 6 When the input–output efficiency of the economic benefit to the private enterprise is higher, the private enterprise tends to invest effort to achieve economic benefits, and the private enterprise may neglect the environmental benefits. This highlights the need to strengthen the incentives related to environmental benefits and enhance the private enterprise's enthusiasm for investing more effort. When the input–output efficiency of the environmental benefit is higher, additional encouragement by the local government and rural residents is not needed. As such, the private enterprise is expected to take the initiative to increase its efforts, which can reduce the need for an incentive.

4 Conclusion and recommendations

Rural environmental governance serves as a pivotal lever for enhancing the quality of life and health of farmers. It also constitutes a core pillar for achieving sustainable development of the rural ecological environment and laying a solid ecological foundation for the advancement of the rural revitalization strategy. Considering the deficiencies in the incentive mechanism of current rural environmental governance, this paper centers on "how to design the agency mechanism through the PPP model to effectively stimulate the enthusiasm of private enterprises in participating" and conducts research. Through the progressive construction of an incentive contract model, the following core conclusions are drawn:

(1) The key to the incentive of the dual-subject model (local government—private enterprise) is clear.

This paper initially constructs a PPP model commissioning and agency incentive contract framework, with the local government serving as the principal and the private enterprise as the agent. The research reveals that, within the dual-governance framework excluding rural resident participation, the willingness and effort level of private enterprises are significantly influenced by their "marginal benefit coefficient". When the marginal benefit coefficient rises, private enterprises, owing to their ability to clearly perceive the positive correlation between input and output, thereby significantly boosting their enthusiasm for participating in environmental governance. This indicates that in the dual model, Enhancing the benefit return mechanism and increasing the marginal benefit coefficient are pivotal strategies for incentivizing private enterprises to proactively engage in environmental governance.

(2) The collaborative value of the tripartite model (local government—rural residents—private enterprises) is distinctly emphasized.

Building on the dual-party model, this paper integrates the role of rural residents to construct a three-party incentive contract model, in which the local government and rural residents serve as principals respectively, while the private enterprise functions as the agent. The research findings uncover two critical patterns: On one hand, when the local government and rural residents assign governance tasks to the private enterprise in their capacities as independent principals, due to inadequate goal coordination given national and decentralized instructions, private enterprises must simultaneously navigate two distinct sets of demand standards. This situation may weaken the overall incentive effect on private enterprises and even result in reduced effort levels due to task conflicts; On the other hand, when local governments and rural residents establish a "coordinated governance" model, the governance efficiency significantly surpasses that of non-cooperative modes. The coordinated framework can not only integrate the resources of both parties (e.g., the government's policy support and residents' grassroots supervision), but also reduce the internal consumption of private enterprises' tasks through a unified goal orientation, thereby creating the prerequisite conditions for effective incentives.

(3) The clear influencing factors and optimization direction of incentives under the three-party collaboration are clarified.

Within the framework of collaborative governance between local governments and rural residents, the research further elucidates the three core factors influencing the incentive effect of private enterprises: Firstly, the degree of risk aversion among private enterprises. The lower the risk aversion, the greater the enterprises' acceptance of rural environmental governance projects, and the more inclined they are to increase long-term investment; Secondly, external environmental uncertainties, such as climate given market fluctuations and policy adjustments, the uncertainty of external variables can alleviate enterprises' concerns about excessive spending on governance costs and unmet return expectations, thereby stabilizing their participation expectations; Thirdly, the governance cost coefficient plays a crucial role. By reducing the unit governance cost through technical and infrastructure support, the enterprise's net income potential can be directly enhanced, thereby strengthening the incentive effect. This means that in the three party collaborative model can form a "multi-factor linkage" incentive force by reducing private enterprises' risk aversion, mitigating external environmental uncertainty, and optimizing cost control, thereby more effectively stimulating their participation enthusiasm.

(4) Collaborative governance represents an inevitable choice for improving the efficiency of rural environmental governance.

Through a comparative analysis of the dual-subject and tri-subject models, it is evident that the efficient progress of rural environmental governance hinges on the collaborative interaction among "local government, rural residents, and private enterprises". In contrast to the binary model's reliance on single incentives and independent commissions, collaborative governance between local governments and rural residents can not only harness policy guidance and resource coordination not only leverage the advantages of the government, but also rely on grassroots supervision and resident feedback to create a governance environment for private enterprises that is "risk controllable, yields clear returns, and reduces costs". This collaborative model not only significantly boosts the participation enthusiasm of private enterprises, but also refines the governance process through the interaction among the three parties, ultimately achieving a simultaneous improvement in both efficiency and quality. Rural environmental governance, and providing strong support for the implementation of the rural ecological sustainable development and the rural revitalization strategy.

In conclusion, the research conclusion of this paper offers a clear direction for the practical optimization of the rural environmental governance PPP model, as evidenced by successful case studies and analyses. Strategic recommendations based on the research are as follows.

Foster rural residents' participation awareness in environmental governance and encourage them to collaborate with local governments as joint principals in implementing incentive contracts for private enterprises. This strategy can improve the efficiency of private enterprises in environmental governance by integrating grassroots participation with governmental supervision. This effectively resolves the incentive dilution issue arising from separate principal roles. Establish special regulatory agencies Enhance and refine the regulatory system to minimize uncertainty factors. For instance, regarding governance standards, various departments may impose distinct requirements. Specialized regulatory bodies can standardize the technical indicators and acceptance criteria for rural environmental governance, preventing private enterprises from being perplexed by ambiguous standards. Local governments ought to enhance systematic oversight to alleviate the adverse effects of external environmental uncertainties on incentive effects. This approach would ensure the stability and effectiveness of private enterprises' participation in environmental governance, as evidenced by the Chinese government's encouragement of such involvement in poverty alleviation and the imperative to engage private firms in national projects. To improve the incentive and constraint mechanisms, differentiated strategies should be implemented for private enterprises with different capabilities. By tailoring incentives to the capabilities and risk preferences of private enterprises, the enthusiasm of all parties can be stimulated, and the effort level of private enterprises can be enhanced. This will maximize the overall governance efficiency.

Author contributions

All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by [Gu Cuiling], [Li Yixian] and [Ma Xiaojian]. The first draft of the manuscript was written by [Zhao Jinhua, Ding Rui] and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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Data availability

No data and material.

Declarations

Competing interests

No conflict of interest exists in the submission of this manuscript, and manuscript is approved by all authors for publication. I would like to declare on behalf of my co-authors that the work described was original research that has not been published previously, and not under consideration for publication elsewhere, in whole or in part. All the authors listed have approved the manuscript that is enclosed.

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