Lellis Thivagar & Priyalatha, *Cogent Mathematics* (2017), 4: 1330180
https://doi.org/10.1080/23311835.2017.1330180

cogent mathematics

PURE MATHEMATICS | RESEARCH ARTICLE

Medical diagnosis in an indiscernibility matrix based on nano topology

M. Lellis Thivagar[1] and S.P.R. Priyalatha[1*]

Received: 30 November 2016
Accepted: 21 March 2017
Published: 18 May 2017

*Corresponding author: S.P.R. Priyalatha, School of Mathematics, Madurai Kamaraj University, Madurai(Dt) 625021, Tamilnadu, India
E-mail: spr.priyalatha@gmail.com

Reviewing editor:
Hari M. Srivastava, University of Victoria, Canada

Additional information is available at the end of the article

Abstract: This paper presents a study of new structure in nano topology. We propose an alternative formulation of nano topological space induced by different neighbourhoods. We also define different types of neighbourhood based on covering of the universe. The properties of various types of neighbourhood such as Right (N_R), Left (N_L), Intersection (N_I) and Union (N_U) of neighbourhoods are discussed. Further, we analysed their indiscernibility matrix and the indiscernibility function which gives the CORE based on nano topology in neighbourhood when applied in real life application.

Subjects: Proofs; Set Theory; Topology; Pure Mathematics

Keywords: covering approximation space; indiscernibility matrix; N_R-neighbourhood; N_L-neighbourhood; N_U-neighbourhood; N_I-neighbourhood; information system

2010 AMS subject classifications: 54B05; 54C05

1. Introduction

The covering approximation spaces can be studied using covering-based rough set theory (Thuan, 2009; Wang, He, Chen, & Hu, 2014; Zhu, 2009), which is a mathematical tool for dealing with data mining, vagueness and granularity in information systems. Lellis Thivagar and Richard (2013) introduced a nano topological space to a subset X of a universe. In this paper, we study the nano topological space approach by using the concept of different types of neighbourhood induced by covering of the univese. We define the covering approximation space stimulated by an arbitrary binary relation

M. Lellis Thivagar

ABOUT THE AUTHORS

M. Lellis Thivagar has published 210 research publications both in national and international journals to his credit. Under his able guidance, 15 scholars have obtained their doctoral degree. In his collaborative work, he has joined hands with intellectuals of highly reputed persons internationally. He serves as a referee for 12 peer reviewed international journals. At present he is the Professor & Chairperson, School of \linebreak Mathematics, Madurai Kamaraj University.

S.P.R. Priyalatha is pursuing PhD under the guidance of Dr M. Lellis Thivagar at the School of Mathematics, Madurai Kamaraj University, Madurai. Four of her research papers published/accepted in the reputed international peer-reviewed journals.

PUBLIC INTEREST STATEMENT

M. Lellis Thivagar introduced nano topological space on a subset X of a universe, defined as lower and upper approximations of *X*. The elements of a nano topological space are called the nano-open sets. But certain nano terms are satisfied just to mean very small, for example nano silver particle. The topology recommended here is named so because of its size, since it has at most five elements in it. The purpose of this paper is to introduce a new structure of nano topology induced by different types of neighbourhood which is right, left neighbourhood and union, intersection of neighbourhood based on covering of the universe. Further, we developed an algorithm to find the core of complete information system by using the indiscernibility matrix. This algorithm is defined in nano topology induced by different neighbourhoods and applied to analyse the real life problems. We believe that the concept of different neighbourhoods can be applied for the study of graph and group theories in nano topological space.

on a triple ordered pair of $(\mathcal{U}, R, C)$. We also define, different types of neighbourhoods (N_j for each $j = R, L, I, U$), such as right neighbourhood (N_R), left neighbourhood (N_L), union of neighbourhood (N_U), intersection of neighbourhood (N_I). Properties and relationship among these neighbourhoods are investigated. Skowron and Rauszer (1992) first proposed to represent knowledge in the form of discernibility matrices in the information system. Now the concept of discernibility matrix has been defined in different types of reduction algorithms for inconsistent information systems (Lashin & Medhat, 2005; Skowron & Rauszer, 1992; Wang et al., 2014). Further, we find the indiscernibility matrix and indiscernibility function, which gives the CORE based on nano topology induced by neighbourhood. Besides, we provide an example for a better understanding of the subject.

2. Preliminaries

Definition 2.1 (Zhu, 2009) For the pair of approximation space $(\mathcal{U}, R)$, where $\mathcal{U}$ is the non-empty finite set of objects called the universe, R be any binary relation on $\mathcal{U}$. Then

- The after set (or) successor of $x \in \mathcal{U}$ denoted by xR (or) $R_s(x)$, where $R_s(x)$ (or) (x) R = $\{y \in \mathcal{U} | xRy\}$.
- The fore set (or) predecessor of $x \in \mathcal{U}$ denoted by Rx (or) $R_p(x)$, where $R_p(x)$ (or) $R(x) = \{y \in \mathcal{U} | yRx\}$.

Definition 2.2 (Thuan, 2009) Let $\mathcal{U}$ be the non-empty finite set of objects called the universe and R be equivalence relation on $\mathcal{U}$. Then the pair $(\mathcal{U}, R)$ is called approximation space.

Definition 2.3 (Mohanty, 2010; Thuan, 2009) Let $\mathcal{U}$ be a non-empty finite set, $\mathcal{C} = \{C_k | k \in K\}$ a family of subsets of $\mathcal{U}$. If none subsets in $\mathcal{C}$ is empty and $\bigcup_{k \in K} C_k = \mathcal{U}$, then $\mathcal{C}$ is called covering of $\mathcal{U}$. The pair $(\mathcal{U}, \mathcal{C})$ is called covering approximation space, if $\mathcal{C}$ is a covering of $\mathcal{U}$.

Definition 2.4 (Pawlak, 1982) Let $\mathcal{U}$ be a non-empty finite set of objects called the universe R be an equivalence relation on $\mathcal{U}$ named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathcal{U}, R)$ is said to be the approximation space. Let $X \subseteq \mathcal{U}$.

(i) The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in \mathcal{U}} \{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.

(ii) The Upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \bigcup_{X \in \mathcal{U}} \{R(x) : R(x) \cap X \neq \emptyset\}$, where $R(x)$ denotes the equivalence class determined by x.

(iii) The Boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.5 (Lellis Thivagar & Richard, 2013) Let $\mathcal{U}$ be the universe, R be an equivalence relation on $\mathcal{U}$ and $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq \mathcal{U}$. Then, $\tau_R(X)$ satisfies the following axioms:

(i) $\mathcal{U}$ and $\emptyset \in \tau_R(X)$.

(ii) The union of elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on $\mathcal{U}$ called as the nano topology on $\mathcal{U}$ with respect to X. We call $\{\mathcal{U}, \tau_R(X)\}$ as the nano topological space.The elements of $\tau_R(X)$ are called as nano-open sets

Definition 2.6 (Lellis Thivagar & Richard, 2015) If τ_R is the nano topology on $\mathcal{U}$ with respect to X, then the set $\beta_R = \{\mathcal{U}, \emptyset, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.7 (Lellis Thivagar & Sutha Devi, 2016) Let $(\mathcal{U}, A)$ be an information system, where A is divided into a set C of condition attributes and a set D of decision attributes. Then a core is a minimal subset of attributes which is such that none of its elements can be removed without affecting the classification powered attributes. And it can be found by $\text{Core}\{A\} = A - \text{Red}_A$.

Throughout this paper, the triple ordered pair of $(\mathcal{U}, R, C)$ is a covering approximation space induced by any binary relation simply called as covering approximation space and binary relation called as relation.

3. Neighbourhoods based on nano topology

This section proposes a new method of different types of neighbourhoods, we call us different neighbourhoods based on nano topology. Also the covering approximation space induced by any binary relation on $\mathcal{U}$, respectively.

Definition 3.1 Let $\mathcal{U}$ be a non-empty finite set and R be a binary relation on $\mathcal{U}$. Then, two different coverings for $\mathcal{U}$ induced from the binary relation R as follows:

(i) Right Covering (briefly, r-cover): $\mathcal{C}_r = \{xR{:}\forall x \in \mathcal{U}\}$ and $\mathcal{U} = \bigcup_{x\in\mathcal{U}} xR$.
(i) Left Covering (briefly, l-cover): $\mathcal{C}_l = \{Rx{:}\forall x \in \mathcal{U}\}$ and $\mathcal{U} = \bigcup_{x\in\mathcal{U}} Rx$.

Definition 3.2 Let $(\mathcal{U}, R, \mathcal{C})$ be the covering approximation space induced by any binary relation R on $\mathcal{U}$. For every element $x \in \mathcal{U}$ and different types of neighbourhoods $N_j(x)$, where $j = R, L, I, U$ as follows:

(i) Right neighbourhood (briefly, R-neighbourhood): $N_R(x) = \bigcap\{K \in \mathcal{C}_r | x \in K\}$.
(ii) Left neighbourhood (briefly, L-neighbourhood): $N_L(x) = \bigcap\{K \in \mathcal{C}_l | x \in K\}$.
(iii) Intersection of neighbourhood (briefly, I-neighbourhood): $N_I(x) = N_R(x) \cap N_L(x)$.
(iv) Union of neighbourhood (briefly, U-neighbourhood): $N_U(x) = N_R(x) \cup N_L(x)$.

LEMMA 3.3 *Let $(\mathcal{U}, R, C)$ be covering approximation space induced by any binary relation and for each $j = R, L, I, U$. If $x \in N_j(y)$ then $N_j(x) \subseteq N_j(y)$.*

Proof

(i) Let $x \in N_R(y)$. Then x is contained in all after set that contains also the element y. Thus $N_R(x) \subseteq N_R(y)$.
(ii) Let $x \in N_R(y)$. And x is contained in all fore set that contains also the element y. Thus $N_L(x) \subseteq N_L(y)$.
(iii) If $x \in N_I(y)$, then $N_I(x) \subseteq N_I(y)$. We can using (i) and (ii), we get $N_R(x) \subseteq N_R(y)$ and $N_L(x) \subseteq N_L(y)$. Then $N_R(x) \cap N_L(x) \subseteq N_R(y) \cap N_L(y)$. Hence $N_I(x) \subseteq N_I(y)$.
(iv) The proof is similar way as in proof (iii). Hence $N_U(x) \subseteq N_U(y)$.

Proposition 3.4 Let $(\mathcal{U}, R, C)$ be covering approximation space induced by any binary relation. Then for each $x \in \mathcal{U}$.

(i) $N_I(x) \subseteq N_R(x) \subseteq N_U(x)$.
(i) $N_I(x) \subseteq N_L(x) \subseteq N_U(x)$.

Proof The proof is directly from the Definition 3.2.

Definition 3.5 Let $\mathcal{U}$ be a non-empty finite set of objects called the universe and R be arbitrary binary relation on $\mathcal{U}$. The triple pair $(\mathcal{U}, R, C)$ is said to be covering approximation space induced by any binary relation. Let $X \subseteq \mathcal{U}$

Table 1. Neighbourhoods

$x \in \mathcal{U}$	a	b	c	d
$N_R(x)$	$\{a\}$	$\{a, b\}$	$\{c, d\}$	$\{c, d\}$
$N_L(x)$	$\{a\}$	$\{b\}$	$\{a, c, d\}$	$\{a, c, d\}$
$N_U(x)$	$\{a\}$	$\{a, b\}$	$\{a, c, d\}$	$\{a, c, d\}$
$N_I(x)$	$\{a\}$	$\{b\}$	$\{c, d\}$	$\{c, d\}$

(i) $L_{N_j}(X) = \bigcup_{x \in \mathcal{U}} \{N_j(x):N_j(x) \subseteq X\}$.
(ii) $U_{N_j}(X) = \bigcup_{x \in \mathcal{U}} \{N_j(x):N_j(x) \cap X \neq \emptyset\}$.
(iii) $B_{N_j}(X) = U_{N_j}(X) - L_{N_j}(X)$.

Definition 3.6 Let $\mathcal{U}$ be universe, $N_j(x)$ be different types of neighbourhoods where $j = R, L, I, U$ and $\tau_{N_j}(X) = \{\mathcal{U}, \emptyset, L_{N_j}(X), U_{N_j}(X), B_{N_j}(X)\}$ forms a nano topology on $\mathcal{U}$ with respect to X. We call $\{\mathcal{U}, \tau_{N_j}(X)\}$ as the nano topology induced by different neighbourhoods.

Example 3.7 Let $\mathcal{U} = \{a, b, c, d\}$ and $X = \{a\} \subseteq \mathcal{U}$, $R = \{(a, a), (a, b), (b, c), (b, d), (c, a), (d, a)\}$. Then $aR = \{a, b\}$, $bR = \{c, d\}$, $cR = \{a\}$, $dR = \{a\}$. Also $Ra = \{a, c, d\}$, $Rb = \{a\}$, $Rc = \{b\}$, $Rd = \{b\}$. Then we get following Table 1.

Hence $\tau_{N_R}(X) = \{\mathcal{U}, \emptyset, \{a\}, \{a, b\}, \{b\}\}$, $\tau_{N_L}(X) = \{\mathcal{U}, \emptyset, \{a\}, \{a, c, d\}, \{c, d\}\}$, $\tau_{N_U}(X) = \{\mathcal{U}, \emptyset, \{a\}, \{b, c, d\}\}$, $\tau_{N_I}(X) = \{\mathcal{U}, \emptyset, \{a\}\}$.

THEOREM 3.8 *Let $(\mathcal{U}, \tau_{N_j}(X))$ be nano topological space induced by different neighbourhoods on $\mathcal{U}$ with respect to X where $X \subseteq \mathcal{U}$. Let $X, Y \subseteq \mathcal{U}$. Then*

(i) $L_{N_j}(X) \subseteq X \subseteq U_{N_j}(X)$
(ii) $L_{N_j}(X) = U_{N_j}(X) = \mathcal{U}$.
(iii) $L_{N_j}(\emptyset) = U_{N_j}(\emptyset) = \emptyset$
(iv) If $X \subseteq Y$ then $L_{N_j}(X) \subseteq L_{N_j}(Y)$, and $U_{N_j}(X) \subseteq U_{N_j}(Y)$.
(v) $L_{N_j}(X) = [U_{N_j}(X^c))]^c$, where X^c is the complement of X.
(vi) If $U_{N_j}(X) = [L_{N_j}(X^c))]^c$, where X^c is the complement of X.
(vii) $L_{N_j}(L_{N_j})(X) = L_{N_j}(X)$

Proof

(i) The proof (i) , (ii) and (iii) is proved by directly from Definition 3.6.
(iv) Let $X \subseteq Y$, $x \in L_{N_j}(X)$. Then $x \in X$ and $N_j(X) \subseteq X$, which means that $x \in Y$ and $N_j(x) \subseteq Y$. Thus $x \in L_{N_j}(Y)$, which implies that $L_{N_j}(X) \subseteq L_{N_j}(Y)$. By the similar way as in $U_{N_j}(X) \subseteq U_{N_j}(Y)$.
(v) If $[U_{N_j}(X^c)]^c = [\{x \in \mathcal{U} | N_j(x) \cap X^c = \emptyset\}]^c$, which is equal to $\{x \in \mathcal{U} | N_j(x) \cap X^c = \emptyset\} = \{x \in X | N_j(x) \subseteq X\} = L_{N_j}(X)$.
(vi) The proof follows from the above (v).
(vii) First, it is clear that $L_{N_j}(L_{N_j})(X) \subseteq L_{N_j}(X)$. Now let $N_j(x) \in (L_{N_j})(X)$. Then $x \in X$ and $N_j(x) \subseteq X$. We must prove that $x \in L_{N_j}(X)$ and $N_j(x) \subseteq L_{N_j}(X)$ as follows: Let $z \in N_j(x)$, then $N_j(z) \subseteq N_j(x)$ by using Lemma 3.3, which implies that $N_j(z) \subseteq X$. Thus $z \in L_{N_j}(X)$ and this means that $N_j(x) \subseteq L_{N_j}(X)$ and then $L_{N_j}(X) \subseteq L_{N_j}(L_{N_j})(X)$. Hence $L_{N_j}(X) = L_{N_j}(L_{N_j})(X)$.

Proposition 3.9 Let $(\mathcal{U}, \tau_{N_j}(X))$ be nano topological space induced by different neighbourhoods on $\mathcal{U}$ with respect to $X \subseteq \mathcal{U}$. Let $X, Y \subseteq \mathcal{U}$. Then

(i) $L_{N_j}(X \cap Y) = L_{N_j}(X) \cap L_{N_j}(Y)$.

(ii) $U_{N_j}(X \cup Y) = U_{N_j}(X) \cup U_{N_j}(Y)$.

(iii) $L_{N_j}(X) \cup L_{N_j}(Y) \subseteq L_{j_c}(X \cup Y)$.

(iv) $U_{j_c}(X \cap Y) \subseteq U_{j_c}(X) \cap L_{j_c}(Y)$.

Proof

(i) Let $N_j(x) \subseteq (L_{N_j}(X) \cap L_{N_j}(Y))$, then $N_j(x) \in L_{N_j}(X)$ and $N_j(x) \in L_{N_j}(Y)$. Thus $N_j(x) \in X$, $N_j(x) \subseteq X$ and $N_j(x) \in Y$, $N_j(x) \subseteq Y$, which means that $N_j(x) \in X \cap Y$, $N_j(x) \subseteq X \cap Y$. Then $x \in L_{N_j}(X \cap Y)$ and this implies $L_{N_j}(X) \cap L_{N_j}(Y) \subseteq L_{N_j}(X \cap Y)$. Now, let $N_j(x) \in L_{N_j}(X \cap Y)$, then $N_j(x) \in (X \cap Y)$ and $x \in X \cap Y$. Thus $x \in X$, $N_j(x) \subseteq X$, $x \in Y$, $N_j(x) \subseteq Y$, which implies that $N_j(x) \in L_{N_j}(X)$ and $N_j(x) \in L_{N_j}(Y)$. Then $N_j(x) \in L_{N_j}(X) \cap L_{N_j}(Y)$, and thus $L_{N_j}(X \cap Y) \subseteq L_{N_j}(X) \cap L_{N_j}(Y)$. Hence $L_{N_j}(X \cap Y) = L_{N_j}(X) \cap L_{N_j}(Y)$.

(ii) The proof is similar way as in proof (i). Hence $U_{N_j}(X \cup Y) = U_{N_j}(X) \cup U_{N_j}(Y)$.

(iii) Since $X \subseteq X \cup Y$ and $Y \subseteq X \cup Y$. Then $L_{N_j}(X) \subseteq L_{N_j}(X \cup Y)$ and $L_{N_j}(Y) \subseteq L_{N_j}(X \cup Y)$. Thus $L_{N_j}(X) \cup L_{N_j}(Y) \subseteq L_{N_j}(X \cup Y)$.

(iv) The proof is similar way as in proof (iii). Hence $U_{N_j}(X \cap Y) \subseteq U_{N_j}(X) \cap U_{N_j}(Y)$.

4. Relationship between the covering approximation space

In this section, we study relationship between the arbitrary binary relation based on covering approximation space $(\mathcal{U}, R, C)$ induced by any relation, respectively.

Proposition 4.1 Let $(\mathcal{U}, \tau_{N_j}(X))$ be nano topological space induced by different neighbourhoods on $\mathcal{U}$ with respect to $X \subseteq \mathcal{U}$. Let $X \subseteq \mathcal{U}$. Then

(i) $L_{N_U}(X) \subseteq L_{N_R}(X) \subseteq L_{N_I}(X)$.

(ii) $L_{N_U}(X) \subseteq L_{N_L}(X) \subseteq L_{N_I}(X)$.

(iii) $U_{N_I}(X) \subseteq U_{N_L}(X) \subseteq U_{N_U}(X)$.

(iv) $U_{N_I}(X) \subseteq U_{N_R}(X) \subseteq U_{N_U}(X)$.

Proof

(i) Let $x \in L_{N_U}(X)$, then $x \in X$ and $N_U(x) \subseteq X$. Thus $x \in X$ and $N_R(x) \subseteq X$, which implies that $x \in L_{N_R}(X)$. Hence, $L_{N_U}(X) \subseteq L_{N_R}(X)$. Also, if $x \in L_{N_R}(X)$ then $x \in X$ and $N_R(x) \subseteq X$, which means that $x \in X$ and $N_I(x) \subseteq X$. Hence $x \in L_{N_I}(X)$ which implies $L_{N_R}(X) \subseteq L_{N_I}(X)$.

(ii) The proof (ii), (iii) and (iv) is similar way as in proof (i).

Proposition 4.2 If $\tau_{N_j}(X)$ is the nano topology based on different neighbourhoods on $\mathcal{U}$ with respect to X. Then the following properties are hold:

(i) $B_{N_I}(X) \subseteq B_{N_R}(X) \subseteq B_{N_U}(X)$.

(ii) $B_{N_I}(X) \subseteq B_{N_L}(X) \subseteq B_{N_U}(X)$.

Proof

(i) The proof (i) and (ii) is the similar way as in Proposition 4.1

Example 4.3 Let $\mathcal{U} = \{a, b, c, d\}$ and $R = \{(a, d), (b, b), (b, c), (c, b), (d, a), (d, c)\}$. Then $aR = \{d\}$, $bR = \{b, c\}$, $cR = \{b\}$, $dR = \{a, c\}$. Also $Ra = \{d\}$, $Rb = \{b, c\}$, $Rc = \{b, d\}$, $Rd = \{a\}$, then we get as $N_R(a) = \{a, c\}$, $N_R(b) = \{b\}$, $N_R(c) = \{c\}$, $N_R(d) = \{d\}$, $N_L(a) = \{a\}$, $N_L(b) = \{b\}$, $N_L(c) = \{b, c\}$, $N_L(d) = \{d\}$, $N_U(a) = \{a, c\}$, $N_U(b) = \{b\}$, $N_U(c) = \{b, c\}$, $N_U(d) = \{d\}$. Let $X = \{c, d\}$. Then we get the following Table 2.

Table 2. Any relation based on $L_{N_j}(X)$, $U_{N_j}(X)$

$X \subseteq \mathcal{U}$	$L_{N_U}(X)$	$L_{N_R}(X)$	$L_{N_L}(X)$	$L_{N_I}(X)$	$U_{N_I}(X)$	$U_{N_R}(X)$	$U_{N_U}(X)$	$U_{N_L}(X)$
$\{a, d\}$	$\{d\}$	$\{d\}$	$\{c, d\}$	$\{c, d\}$	$\{c, d\}$	$\{a, c, d\}$	$\{a, b, c, d\}$	$\{b, c, d\}$

Table 3. $B_{N_j}(X)$ induced by any relation

$X \subseteq \mathcal{U}$	$B_{N_I}(X)$	$B_{N_R}(X)$	$B_{N_U}(X)$	$B_{N_L}(X)$
$\{a, d\}$	$\emptyset$	$\{a, c\}$	$\{a, b, c\}$	$\{b\}$

Example 4.4 Consider Example 4.3 and to find the neighbourhood of boundary region as follows (Table 3):

5. Indiscerniblity matrix for the basis in $\tau_{N_R}(X)$

In this section, we study the indiscernibility matrix of the basis $B = \beta_{N_R}$ of the nano topology induced by right neighbourhood. We study the information table giving the information for six patients regarding their diabetes.

Definition 5.1 Let $(\mathcal{U}, A)$ be an information system, where $\mathcal{U}$ is a universe and A is divided into a set C of condition attributes and a set D of decision attributes, defines a matrix $M(B)$ called indiscernibility matrices. Let $B \subseteq A$ and each entry $M(B)(x_i, x_j) \subseteq A$ consists of a set of attributes that can be used to indiscern between objects $x_i, x_j \in \mathcal{U}$. That is $M(B)$ defined as $[c_{ij}] = \{a \in B{:}a(x_i) = a(x_j)\}$ for $i, j = 1, 2$ and $M(B)$ is an 2×2 matrix or 1×1 matrix other than $\mathcal{U}$ from the basis $B = \beta_{N_R}$ and $M(B)$ assigns to each pair of objects x_i and x_j subset of attributes B.

Definition 5.2 Let $(\mathcal{U}, A)$ be an information system. Then a core is the set of all single element entries of the indiscernibility matrix $M(B)$. That is $\text{CORE}(B) = \{a \in B{:}[c_{ij}] = (a), \text{for some } i, j\}$.

Definition 5.3 Let $(\mathcal{U}, A)$ be an information system and an indiscernibility function $F(B)$ is a Boolean function of the attributes $B \subseteq A$ and defined as follows: $F(B) = \wedge\{\vee[c_{ij}]{:} \text{for } i, j = 1, 2\}$ where $\vee[c_{ij}]$ is the disjunction operation on c_{ij} and $\vee$ denotes the Boolean sum $(+)$ or $(\cup)$ and the $\wedge$ denotes the Boolean multiplication $(\cdot)$ or $(\cap)$.

6. Algorithm

Step I: Given a finite universe $\mathcal{U}$, a finite set A of attributes that is divided into two classes, C of condition attributes and D of decision attributes and any binary relation R on $\mathcal{U}$ corresponding to C and a subset X of $\mathcal{U}$, represent the data as an information table, columns of which are labelled by attributes, rows by objects and entries of the table are attribute values.

Step II: Find the neighbourhood of lower approximation, neighbourhood of upper approximation and the neighbourhood of boundary region of X.

Step III: Generate the nano topology induced by different neighbourhood $\tau_{N_R}(X) = \{\mathcal{U}, \emptyset, L_{N_R}(X), U_{N_R}(X), B_{N_R}(X)\}$ with respect to X and its basis $\beta_{N_R}(X) = B = \{\mathcal{U}, L_{N_R}(X), B_{N_R}(X)\}$.

Step IV: From the indiscernibility matrix $M(B) = [c_{ij}] = \{a \in B{:}a(x_i) = a(x_j)\}$ for $i, j = 1, 2$ and $M(B)$ is an 2×2 matrix or 1×1 matrix otherthan $\mathcal{U}$ from the basis $B = \beta_{N_R}$.

Step V: Find the indiscernibility function $F(B)$ which gives the CORE (Figure 1).

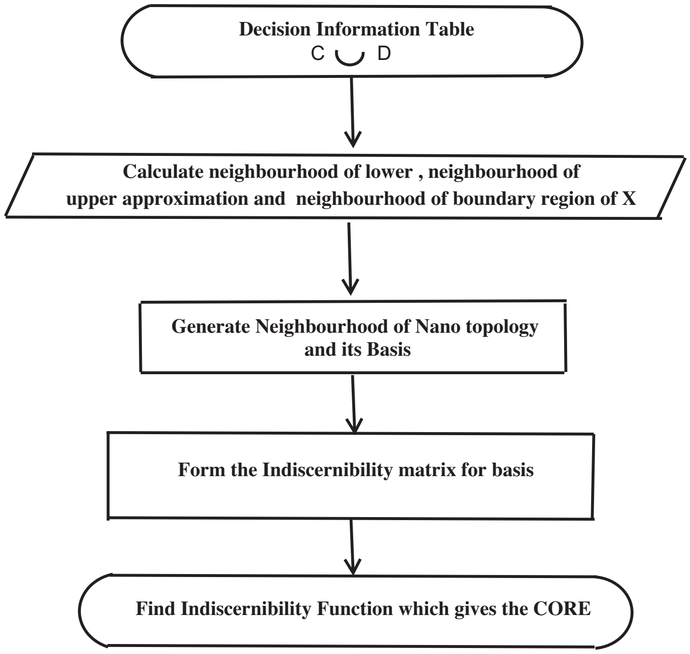

Figure 1. The flow chart of algorithm to find the CORE of information system.

Example 6.1 Consider the following information table (or) decision table giving information about the patients' data-set and respective possible symptoms are regarding their Diabetes. Diabetes is a group of metabolic diseases in which a person has high blood sugar, either because the body does not produce enough insulin, or because cells do not respond to the insulin that is produced. Though both men and women can be affected by this disease, the rate of diabetes in women has increased considerably in the recent years. Moreover, it is said that women are more at risk of being affected by health problems caused by diabetes than men. If one experience *blurred vision* suddenly then she should consult a physician, since high blood glucose levels may affect our eyes directly. *Unexplained weight loss or gain* is one of the common sign of diabetes in women and men. Another symptom is *frequent urination*. The human body tries to get rid of excess sugar through urine and hence, one feels the need to urinate often. In diabetes, glucose in the blood cannot move in to cells, so it stays in the blood. This not only harms the cells that need the glucose for fuel, but also harms certain organs and tissues exposed to the high glucose levels. This high blood sugar produces the classical symptoms of frequent urination and is medically called Polyuria. As excessive urination not only eliminates the extra sugar present in the body, but also large amounts of water, the individual may suffer from the problem of dehydration. Due to this, she may also experience *excessive thirst* and is medically know as Polydipsia throughout the day which is another symptom of diabetes in women. A *feeling of itchiness* on your skin is sometimes a symptom of diabetes. The following Table 4 gives information of six patients who consult a doctor with one or other of the symptoms of Diabetes.

Table 4. Patients with possible symptoms

Patient	Vision (V)	Weight (W)	Frequent urination (U)	Excessive thirst (T)	Itchy skin (I)	Diabetes
$\mathcal{P}_1$	Blurred	Gain	Yes	No	No	Yes
$\mathcal{P}_2$	Normal	Normal	Yes	Yes	Yes	No
$\mathcal{P}_3$	Blurred	Loss	Yes	Yes	No	Yes
$\mathcal{P}_4$	Blurred	Loss	Yes	Yes	No	Yes
$\mathcal{P}_5$	Blurred	Gain	No	No	Yes	No
$\mathcal{P}_6$	Blurred	Normal	No	Yes	Yes	No

Table 5. The indiscernibility matrix

	$\mathcal{P}_1$	$\mathcal{P}_3$	$\mathcal{P}_4$
$\mathcal{P}_1$	*At*	*V*, *U*	*V*, *U*
$\mathcal{P}_3$	*V*, *U*	*At*	*V*, *W*, *U*
$\mathcal{P}_4$	*V*, *U*	*V*, *U*	*At*

Step 1: Let $\mathcal{U} = \{P_1, P_2, P_3, P_4, P_5, P_6\}$ be a set of six patients taken with the possible symptoms of diabetes. The set of conditional attributes is represented by C = {Vision, Weight, Frequent urination (U), Excessive Thirst (T), Itchy skin (I)} $C = \{\text{Vision}, \text{Weight}, \text{Frequent urination}\,(U), \text{Excessive Thirst}\,(T), \text{Itchy skin}\,(I)\}$ and the set D represented the decision attribute, where $D = \{\text{Diabetes}\}$. If $R = \{\text{Vision}, \text{Weight}, \text{Frequent urination}(U), \text{Excessive thirst}(T), \text{Itchy skin}(I)\}$ $= \{(P_1, P_1), (P_2, P_2), (P_3, P_3), (P_4, P_4), (P_5, P_5), (P_6, P_6), (P_3, P_4), (P_4, P_3)\}$ is any binary relation on $\mathcal{U}$. Then $N_R(P_1) = \{P_1\}, N_R(P_2) = \{P_2\}, N_R(P_3) = \{P_3, P_4\}$, $N_R(P_4) = \{P_3, P_4\}, N_R(P_5) = \{P_5\}, N_R(P_6) = \{P_6\}$.

Step 2: Let $X = \{P_1, P_3, P_4\}$. Then $L_{N_R}(X) = \{P_1, P_3, P_4\}$ and $U_{N_R}(X) = \{P_1, P_3, P_4\}, B_{N_R}(X) = \emptyset$. Hence $\tau_{N_R}(X) = \{\mathcal{U}, \emptyset, \{P_1, P_3, P_4\}\}$.

Step 3: In $\tau_{N_R}(X) = \{\mathcal{U}, \emptyset, \{P_1, P_3, P_4\}\}$ and its basis $\beta_{N_R}(X) = \{\mathcal{U}, \emptyset, \{P_1, P_3, P_4\}\}$. Here the basis consists on only one element other than $\mathcal{U}$ and it is denoted by $\{P_1, P_3, P_4\}$.

Step 4: The indiscernibility matrix $M(B)$ for the basis is given by (Table 5).

Step 5: The indiscernibility function $F(B) = \{V + U\} \cdot \{V + W + U\} = V + U$ where $+$ denotes the Boolean sum $(\vee)$ or $(\cup)$ and the $\cdot$ denotes the Boolean multiplication $(\wedge)$ or $(\cap)$. Hence the $\text{CORE} = \{U, V\}$.

Observation: Based on the algorithm, we have found the CORE = $\{\text{Frequent urination}(U), \text{Blurred vision}(V)\}$ are the key attributes that have close connection to the diabetic patients.

7. Conclusion

This paper introduced a nano topology induced by different neighbourhoods. An algorithm was developed to find the indiscernibility matrix which gives the CORE in an information system. The indiscernibility matrix was applied in an information system to identify the core in a patient data-set based on their symptoms. Further, our result suggests the possibility of extending the indiscernibility matrix to the incomplete information system to find the CORE.

Funding
This work was supported by University Grants Commission, New Delhi.

Author details
M. Lellis Thivagar[1]
E-mail: mlthivagar@yahoo.co.in
ORCID ID: http://orcid.org/0000-0001-5997-5185
S.P.R. Priyalatha[1]
E-mail: spr.priyalatha@gmail.com
ORCID ID: http://orcid.org/0000-0001-6143-7407
[1] School of Mathematics, Madurai Kamaraj University, Madurai(Dt) 625021, Tamilnadu, India.

Citation information
Cite this article as: Medical diagnosis in an indiscernibility matrix based on nano topology, M. Lellis Thivagar & S.P.R. Priyalatha, *Cogent Mathematics* (2017), 4: 1330180.

References

Lashin, E. F., & Medhat, T. (2005). Topological reduction of information systems. *Choas Solutions and Fractals, 25*, 277–286.

Lellis Thivagar, M., & Richard, C. (2013). On nano forms of weakly open sets. *International Journal Mathematics and Statistics Invention, 1*, 31–37.

Lellis Thivagar, M., & Richard, C. (2015). On nano continuity in a strong form. *International Journal of Pure and Applied Mathematics, 101*, 893–904.

Lellis Thivagar, M., & Sutha Devi, V. (2016). Computing technique for recruitment process via nano topology. *Sohag Journal of Mathematics, 3*, 37–45.

Mohanty, D. (2010). Rough set on generalized covering approximation spaces. *International Journal of Computer Science and Research, 1*, 43–49.

Pawlak, Z. (1982). Rough sets. *International Journal of Computer and Information Sciences, 11*, 341–356.

Skowron, A., & Rauszer, C. (1992). The discernibility matrices and functions in information systems. In S. Aski (Ed.), *Intelligent decision support, theory and decision library* (Vol. 11, pp. 331–362). Netherlands: Springer.
Thuan, N. D. (2009). Covering rough sets from a topological point of view. *International Journal of Computer Theory and Engineering, 1*, 1793–8201.
Wang, C., He, Q., Chen, D., & Hu, Q. (2014). A novel method for attribute reduction of covering decision systems. *Information Sciences, 254*, 181–196.
Zhu, W. (2009). Relationship between generalized rough sets based on binary relation and covering. *Journal of Information Science, 179*, 210–225.